THE

BAKHSHĀLĪ MANUSCRIPT

A Study in Mediæval Mathematics

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THE BAKHSHALI MANUSCRIPT

Early Hindu Mathematics

A Study in Mediaeval Mathematics

KAY G.R.

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PREFACE.

In order to correct an impression that certain passages in this volume might convey unless distinctly qualified, I must here refer to my indebtedness to the late Dr. Hoernle. Indeed, a considerable part of the analysis of the MS. is really his work,* and by his preliminary survey of the manuscript my task was considerably lightened. It was at Dr. Hoernle's special request that I undertook to carry on the work he had started, and he handed over to me most of the material he had himself prepared. Had he lived a little longer I should, no doubt, have had the benefit of further help from him, and this volume might have been issued as our joint work. Dr. Hoernle's lamented death prevented that plan being carried out; and unfortunately my views are so often opposed to those that where held by Dr. Hoernle that it would hardly be proper to make him a participator in them.

I am much indebted to Bodley's Librarian for special facilities that enabled me to examine the original manuscript under the most favourable conditions; to the Oxford University Press for their most excellent work in preparing the photographs of the manuscript and the collotype reproductions of the text; and to the Manager, Government of India Press, Calcutta, for the care and skill with which the transliteration has been printed.

G. R. KAYE.

BANHAM,
Attleborough,
Norfolk.

* Sections B, G, H, K and L are almost wholly the work of Dr. Hoernle, who also transiterated about half of the leaves of the MS. References to his published papers on the MS. are given on page 2.

PART III.

1.—The Text Re-arranged.

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Quotations in the text are distinguished by daggers† . . . . . †, and abbreviations by ° superscribed. Asterisks attached to numbers denote change-ratios (See § 103). In the foot-notes angular crotchets < > indicate that the portion enclosed formed part of the argument or was implied in the original text, but is now missing.

On pp. 13 and 14 of Part I are tables equating the Bodleian Library order with the revised arrangements.

The notes attached to the revised arrangement are very crude and are presented with considerable diffidence; but they are the result of much labour and will possibly save the student of the MS a good deal of spade work.

G. R. K.

Owing to Mr. Kaye's unfortunate death, the last proofs of this part have been prepared for the press by Mr. K. N. Dikshit, Deputy Director General of Archaeology for Exploration, who has also made a few emendations.

A 1.

. . . yatra y . g . . . ' . bhāgaṁ chaiva kārayet kshetra vaipulya . . . 40ᵃ recto.

. . pṛishṭhā śata-dvayaṁ chaiva uchare śatam ekataḥ vaipulyād vi . . . .

. . śa dvādaśa nṛi śakas tathā | sapta paṁcha bhavet chānaṁ bhakti sthāne 39ᵃ recto.

. . r . . . . dhā sapta paṁchānāṁ tṛi-dvi meka ϕ prakal*p*itaṁ | tasya

vāhasya kiṁ ka . . . . . tatrā mama | kshetrasya.

sthāpanaṁ kṛiyate |

kshetraṁ 100

. . . .	15	12	10	7	5

karanaṁ | kshetra . .

300 . . *d*aṁ cha . . . 38ᵇ recto.

vaipulyād yogaṁ . . .

esha shaṭ . . . . . 39ᵇ recto.

. . . . . chaiva tat phal . . . . . guṇitā jātā 40ᵃ verso.

6210 esha vāhasya kāṇḍa pramāṇaṁ . . . śake mūlyaṁ *kartavy*aṁ |

adha chchhedam chatus sashṭhi . . . . la | *sutha dvi* triṁśa*bhi* maṇḍalakai 39ᵃ verso.

tallika esa chchhedaṁ bhavati . . . yathe chchh . . kāryā | *s*utha ṭala

kṛiyā udāharanaṁ . . ṭalasya . . . mekaṁ ta dvā sashṭi śatānāṁ

daśādhīkānāṁ kiṁ mulyaṁ : ṭalla . . . tale a° 38ᵇ verso.

| 1 . . rū 1 mūlye | 6210 maṇale abhim | pha . . .

| 1

[38-40.] Plates xxvi and xxvii exhibit some sixteen fragments all placed out of order. Some of these have now been pieced together. (See the illustration facing page 4.)

This grouping is not final because some of the fragments consist of portions of two or more leaves stuck together, and until these are separated no exact order can be achieved.

We should naturally expect the first leaves of the manuscript to be comparatively more damaged than those in the middle of the book, and the 'find order' and the writing indicate that these fragments are probably portions of early leaves but neither of these criteria is rigorous and it is quite possible that we have placed the fragments in their wrong places.

[38-40 *recto.*] These fragments appear to relate to a geometrical problem concerning an area whose width (*vaipulya*) is increased.

[38-40 *verso.*] A fragment of a problem connected with the area of a circle or the surface (*tala*) of a sphere. The phrase *esha vāhasya kāṇḍa pramāṇam* ought to be illuminating but is not. The change-ratio 64 is possibly connected with a "square measure." See Part I §108(*b*). The number 6210 = $3^3 \cdot 230$ is said to be the product of certain quantities.

A 2.

. . . . . **ksh** . . . . . daśa | chatur-daśa tṛitī- 39[b] recto.

yasya chaturthasya . . . . . bhāgās tasyaiva paṁchama . . . 40[a] recto.

. . . *bh*āgā viṁśas cha dasagunā | saptama ksh . . . jñā . . . . . 38[a] recto.

yaṁ śataṁ | sarve miśrāpi dṛishṭhaṁ cha śatāni . . . 40[b] recto. 39[d] recto.

. dhanaṁ 1 . . . . . 10 || esha ekaika bhāgā guṇitā jā . . . . 39[b] verso.

60 | 180 | 200 | 300 | evaṁ dhanaṁ 1200 pratyaya trairāsikena . 00 40[c] verso.

. . . dhanaṁ 1200

.	pha° 144	38[a] verso.
.	pha° 16.	39[d] verso.
.	pha° 180	40[b] verso.
20	pha° 200	

A 2. [38-40.] The writing on the two sides differs (*recto* α_1, *verso* α_2) and there are other indications that the fragments consist of portions of two leaves at least.

A 3.

bdhāṁbupayaso ghaṭaḥ eka miśrikṛit . . . 40[d] recto.

karaṇam | havya tulyaṁ vinikshipyaḥ 40[a] recto.

4	5	6	kuru prakshepakaṁ tata praksh
4	5	6	
4	5	6	

A 3. [38-40 recto.] See the plate facing page 4. The meaning is not clear, but $x(x+y+z)=60$, $y(x+y+z)=75$, $z(x+y+z)=90$, whence $(x+y+z)^2=225$ and $x+y+z=15$. The answers are $x=4$, $y=5$, $z=6$.

The writing is classed as α_2.

A 3—*contd.*

sthāpya | 4 pa | 5 aṁ | 6 | 39ᵇ recto.
15 | 15 | 15

4 | 5 | 6
15 | 15 | 15

4 | 5 | 6
15 | 15 | 15

. . . . . kṛiyate || chaturbhi pañchabhish shaḍbhi g . . prathama

rāśi yoga | 60 | vartyaṁ 4 madhū ghaṭa . . dvitīya pañktyā yoga | 75 | 38ᵃ recto.
15 | 15

vartyaṁ | 5 | pānīyaṁ || tritīya pañktya kṛiyate yogaṁ | 90 | vartyaṁ jātaṁ
1 | 15

6 payasāṁ . . . . .
1

. . . . . kṛitvā . . guṇetu || eko . . . kṛitaṁ 40ᵃ verso.

śatatrayaṁ pañchabhi ✓ purushair labdhaṁ kiṁ ādyaṁ prathamaṁ dhanaṁ ||

. . . . . 120 || . 2257 . . ṇaṁ . . . . . t śeshe kshepa

16 anenātra bhāga | 32 | labdha | 2 | 40 | pha . . . . 39ᵇ verso.
16 | 12 | pha 120

. . labdher bhāg . | 28 | jātā 14 labdha kshepam . . . . . dṛi 60
2

prakshepa yukti 30 vibhaktaṁ | 1 | . . . ṇitā jātā . . 14 | 18 38ᵃ verso.
30

| 28 | evaṁ 60

[38-40 *verso.*] Five separate fragments, too mutilated to be intelligible.

A 4.

i. . . . . dviguṇaṁ cha tṛi-ūna cha tṛitīyasya dhanaṁ bhavet . . . . 54ᵃ verso.

saṁyutaṁ | eka-viṁshatibhi ɤ . kṛito dīnāraistu rai . . . . . ya

tu daṁ sā pṛithag vachaḥ ||

karaṇaṁ || yasya padaṁ na jñāyate . . . . . etat prathamasya

dhanaṁ |

ii. . . . . cha dattavān hastag . . . . . yeshāṁ | 0 | 2 54ᵇ verso.

| 2+ . . . . . 54ᵃ recto.

dhanaṁ . . .

1	2	4	8
1	1	1	1

yātā | tayor yogaviyo . . . . kṛitāṁ rāshayaḥ

2	1	2+	9+	dṛi° 82
1	1	1	1	
1	2	4	8	
1	1	1	1	

. . bhājyā hitveti | tatra uttara rāśi . . . . . uttaraṁ riṇam jātaṁ

. . . . . .

(b) sūtraṁ || . . . . . (c) jātaṁ 76 esha prathamasya . . . . . 54ᵇ·ᶜ recto.

A 4. [54.] Folio 54 possibly consists of two leaves, or rather fragments of them, for there are ten pieces. The writing on the two sides differs—that on 54 *recto* may be classed as α_1 and that on the left side as α_2 and in this respect the leaf resembles fol. 35ᵇ. There is a characteristic *ye* at the bottom of 54ᵇ *verso* which is also found on 29ᵇ *recto* et *verso*.

[54ᵃ *verso*.] Seems to contain portions of a *sūtra*, an example and solution. The phrase *dviguṇaṁ cha tṛiūna* seems to be referred to on fol. 35ᵃ *recto* but there we have *tryūna* with a particularly noteworthy conjunct *tryū* (see table IV, 5 part ii). The term *hastag(atam)* on 54ᵇ *recto* (not necessarily connected with 54ᵃ) occurs only once more on fol. 1 *recto*.

[54ᵃ *recto*.] The phrase *tayor yogaviyo* also occurs on fol. 35ᵇ *verso*.

A 5.

. . . . kasmāt kāraṇā | tayor yogaviyogasy āviyogas . . . . . 35ᵇ recto.

bhājitā purusha 15 anena bhaktvā dhanaṁ

9
15

. . . . . padvaya

sahitaṁ ||

. . . . . mūleṇa

1
1
2

eta dviguṇaṁ

3

dviyuta . . . . . yasya 35ᵇ verso.

dhanaṁ | tadeva svārdhaṁ

3

asyārdhaṁ

1
1
2

yutaṁ nyāsa

A 5. [35ᵇ.] The writing is different on the two sides (α_1 and α_2) and possibly the fragment is a portion of two leaves stuck together. The phrase *bhājitā purusha* occurs on 51ᵇ *recto*.

A 6.

bhājitā hitvā | tatrottarā 1 | 1 | yutaṁ 2 | 1 3 3 / 4 2 1 | . . . . 51ᵇ recto.

9 | . eshā ϕ . ītha bhājitā | purushaḥ 1 3 3 / 4 2 1 eshāṁ sadṛiśe 35ᵃ recto.

4 / 19 dhanaṁ 19 / 1 anena guṇitaṁ jātaṁ 4 esha prathamasya dhanaṁ

. . . . dviguṇaṁ 12 dvi-yutaṁ | 14 | eta dvitīyasya . . . . .

guṇaṁ 21 dvi-guṇaṁ 42 try-ūṇaṁ 39 eshaḥ nyāsaḥ

pratya . . . . daśam agravṛindānāṁ chatur-daśa ekonachatvāriṁśa | tat

pād-ārdha tṛi-bhāgā . . . . . .

4 1 / 4 | pha° 4 | evam dī° 21 esha prashṇa etair . . .

A 6. [51ᵇ and 35ᵃ *recto*.] The position is uncertain but the writing is of the α_3 style and there are slight indications of connexion with folio 54. Fol. 35ᵇ is in α_1 writing. See the plate facing page 4. (Read 51 *recto* B, not *verso*.)

The fragmentary contents are not clear. We have $1+1=2$; $\frac{1}{2}+\frac{1}{4}+3=\frac{19}{4}$; $\frac{19}{19/4}=4$ and $\frac{2.3.(12+2)}{2}-3=39$

Apparently a fragment of the *sūtra* on which the solution depends is preserved on fol. 54ᵃ *verso*, but the evidence, consisting of the phrase *dviguṇaṁ cha tṛir-ūna*, is slender.

udā || . . . . . | 6 | . . . yoga 111 śeshā ϕ 51ᵇ verso.

purusha bhājitā purushāḥ 1 1 / 4 6 | 1 / 5 eshāṁ sadṛishe yutiṁ kṛitvā yutā 37 / 60 35ᵃ verso.

bhājitā 60 / 37 | . . . . . esha gavāśva mahishī pratyaika śāleshu bhāga . .

1 śā°	180 gā°	1 / 4	phalaṁ 45	. . . .		
1 / 1 śā°	180 / 1 asvā°	1 / 6	phalaṁ 30	4+	26
1 / 1 śālā	180 / 1 mahi°	1 / 5	phalaṁ 36	5+	9

[51ᵇ & 35ᵃ verso.] The writing is of the α_1 class. The 'find order' of folio 51 is 37 while that of 35 is not known. The position is very uncertain. What remains of the problem is

$\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=\frac{1}{1\frac{1}{2}}$ $111\times\frac{1}{1\frac{1}{2}}=180$

1 enclosure : 180 cows :: $\frac{1}{4}$: 45 ? subtract 6=39.

1 „ : 180 horses :: $\frac{1}{6}$: 30 subtract 4= 26.

1 „ : 180 buffaloes :: $\frac{1}{5}$: 36 subtract 5=29 ?

A 7.

51ᵃ recto.

1	1	1	1
1	1	1	1

. . . . | 1 | 2 | 3 | eshāṁ yutā | 6 / 1 | . . . . . | 48 | śeshā ϕ

purusha sa 4 || anena bhājitā-r-labdhā . . . .sya bhavati | 12 | 13 | 14 | 15 | ekatraṁ 54 ||

ii. udā° || kaśchid rājā dade dānaṁ sapta-paṁchāśakaṁ budha |

paṁchā . . . . . pravakshyāmy=anupūrvaśah . . . . .

dvi-guṇa dvi-guṇaṁ chaiva rūpa rūpottare . . . .

. . . prathame prāptaṁ kiṁ prāptaṁ apare jane ||

0	1	2	. . . . .
1	1	1	. . . . .

51ᵃ verso.

					dṛi 329 / 1
1	3	9	27	81	

karaṇaṁ | uttar . . . . . tatrottara rāśīnāṁ yoga 87 esha dhanā dṛishyā śodhanīyā jātā 242 . . . . . | purusha | 1 | 3* | 9 | 27 | 81 | yoga 121 anena . . . . . jātā | 2 | esha dvau prathamasya dhanaṁ ||

2 | 6 | 18 | 54 | 162 | uttara rāśī saṁyutaṁ jātaṁ

2	15	48	147	444	eshāṁ . . . . .
1	2	2	2	2	

A 7. [51ᵃ.] Either there are two leaves stuck together here or there is some over-lapping. The writing on both sides is α_1. The find order is 37.

[51ᵃ recto.] i. There is not enough material for reconstruction but $x+(x+1)+(x+2)+(x+3)=54$ therefore $4x=54-6$ and $x=12$ is indicated.

ii. A certain Rāja makes presents to 87 wise men, etc. See 52 recto.

[51ᵃ verso.] This apparently does not connect up with the other side. It exhibits the solution of an example which may be expressed by

$$\left.\begin{matrix} t_1+3t_1+3^2t_1+3^3t_1 \\ \frac{1}{2}t_1+\frac{1}{2}(t_1+t_2)+\frac{1}{2}(t_1+t_2+t_3)+\frac{1}{2}(t_1+t_2+t_3+t_4) \end{matrix}\right\} =329$$

Set $t_1=2$ then the first series becomes 242 and the second 87 and the combined series is $2+\frac{1}{2}^3+4^3+\frac{1}{2}^7+\frac{1}{2}^9=329$. See also page 47.

[6] Omitted in the MS.

A 8.

i. . . . . 57 . . . tvedaṁ jātaṁ 3 anena chāliṁśa guṇaye 52 recto.

jātā 120 . . . . . vam śurānāṁ ||

pratyaya trai-rāśikena

1	vaṁ	120	1	12	...
1		1	10	1	
1	vaṁ	120	1	15	...
1		1	8	1	
1	vaṁ	120	1	30	...
1		1	4	1	

ii. udā° || dhanā sva-m-ardho saṁśoddhya . . . chottarīyakaṁ |

tat seshā paṁchamo bhāgo . . . . śata dvayaṁ |

aśītyādhikaṁ dhanaṁ chaiva kiṁ ādyaṁ prathamaṁ dhanaṁ ||

. . . . . asya dvayānāṁ śatānāṁ pāda . . . rdhaṁ 52 verso.

śataṁ bhavati 150 atrāpī paṁcha bhāga 30 || evam . . . . . .

1	1	1	pha° piṇḍā 280	
2	4	5	
	1	1	
	2+	4+	
		1	
		2+	

paṁchamī jātī karaṇaṁ kṛita . . . 280 | aṁśa yuti | 28 / 40 | bhaktaṁ | 40 / 28

dhanu 280 *guṇitaṁ jātaṁ* 400 esha phalaṁ bhavati ||

A 8. [52 *recto.*] i. The writing on both sides is α_2 and exhibits examples of the 'sickle-shaped' medial I and I. The 'find order' is 57. It is possible that 52 *recto* gives parts of the solution of the example on 51 *recto* which would make that page the reverse, but I doubt the connexion. What is left of the solution means

$x(\frac{1}{10}+\frac{1}{8}+\frac{1}{4})=57$ or $x\frac{19}{40}=57$ and $x=120$. A proof by the 'rule of three'

$1 : 120 :: \frac{1}{10} : 12$

$1 : 120 :: \frac{1}{8} : 15$

$1 : 120 :: \frac{1}{4} : 30$

ii. The example, which is continued on 52 *verso*, may be expressed by $x(1-\frac{1}{2})(1-\frac{1}{5})(1-\frac{1}{4})=x-280$ whence $x=\frac{280}{38/40}=400$. A proof follows: $\frac{400}{2}=200$ and $400-200=200$; $\frac{400}{4}=50$ and $200-50=150$; $\frac{150}{5}=30$ and $150-30=120$; and $400-120=280$.

Again $\frac{1}{2}+\frac{1}{5}(1-\frac{1}{2})+\frac{1}{4}(1-\frac{1}{2})(1-\frac{1}{5})=\frac{1}{2}+\frac{1}{10}+\frac{3}{20}=\frac{19}{20}$ and $280\times\frac{20}{19}=400$.

A 9.

. . . . . vinirdiset ‖ 29ᵃ recto.

udā° ‖ **dhana** . . . . . .

ādya dvitīya yonmiśraṁ dhanaṁ tatra ttrayodashaḥ

dvitīya ṭṛitīya yonmi . . . . . *chatur*daśa

ādya ṭṛitīya yonmiśraṁ dhanaṁ paṁchadaśa smṛitaḥ

ekaikasya dhanam . . . chchhiche katthyatāṁ mamah

13	14	15
1	1	1

. . . . . prathamaṇ yasya tatrechhā paṁchaḥ | 5 | tat prathama . . . 29ᵇ recto.

. . | 13 | 14 | 15 | tadādiś† śodhayet kramāt † ādi . . . . . eta
| 1 | 1 | 1 |

chatur-daśabhi śodhya śeshaṁ | 6 | etat paṁcha 29ᶜ recto.

. . . . . dvitīya yonmiśraṁ dhanaṁ 29ᵈ verso.

dvitīya ṭṛitīya yonmiśram dhanaṁ sapta-dasha smṛitaḥ

ṭṛitīyaś chaturthayo . . . . . .

chatuϕ paṁchaka miśraṁ tu dhanaṁ ekona-vimśati |

prathama tatra cha

ekaikasya dhanaṁ kiṁssyād vechchhi . . . . .

16	17	18	19	20
1	1	1	1	1

29ᵇ verso.

karaṇaṁ ‖ ichchhā . . . . . dani†sodhayet kramāt† tatrādi 16

śud . . . . . ṭṛitīyāvaṁ śoddhya 7 chaturthāyaṁ śoddhya 12 paṁ 29ᶜ verso.

. . .

A 9. [29.] Folio 29 consists of six fragments, of which only the four larger ones need be considered at present. The correct order is *d*, *b*, *c*. Fragment *b* fits under *d* and *c* under *b* while *a* goes with folio 27. See the plate facing page 4.

[29 *d*, *b*, *c recto.*] The problem and its solution here partly preserved may be represented by $x_1+x_2=13$, $x_2+x_3=14$, $x_3+x_1=15$. If $x_1=5$ then $x_2=8$, $x_3=6$ and $x_3+x_1=11$ and the correct values are found from $x_1=5+\frac{15-11}{2}=7$, $x_2=13-7=6$, $x_3=8$. The phrase "śodhayet kramāt" recurs in the next example and is a quotation from a lost *sūtra*.

[29 *d*, *b*, *c verso.*] The example here given (continued on folio 27 *verso*) is formulated with exactly the same phraseology as the previous one. It may be represented by

$x_1+x_2=16$, $x_2+x_3=17$, $x_3+x_4=18$, $x_4+x_5=19$, $x_5+x_1=20$. If $x_1=10$, $x_2=6$, $x_3=11$, $x_4=7$, $x_5=12$ and $x_3+x_1=22$

Therefore the correct value of x_1 is $10+\frac{20-22}{2}=9$, $x_2=7$, etc.

The phrases *ichchhā* . . . and *śodhayet kramāt* are quotations from a lost *sūtra*.

A 10.

. . . . . masya dhanaṁ | eshaṁ anukkrameṇa 27 verso.

pūrvokt . . . . . .

9 pra°	7 dvi°	10 tṛi°	8 cha°	11 paṁ°
7 dvi°	10 tṛi°	8 cha°	11 paṁ°	9 pra°

yutaṁ jātaṁ pratyaik . . . 16 | 17 | 18 | 19 | 20 . . . . . .

. . . evaṁ sarvatra kārayet ‖ 29a verso.

karaṇaṁ | †pṛithak rūpaṁ vinikshipya† | pṛithak rūpaṁ kshiptaṁ jātaṁ . 27 recto.

†. . . bhyāso† tatra guṇa | 3 | 4 | abhyāsaṁ | 12 | † rūpahīnaṁ† 1

. . abhyāsā chatuϕ paṁchakā | atra kshiptaṁ jātaṁ | 15 | 16 |

eśa triguṇ . . . . *tā* mūla . . ni chatuϕ paṁcha | 5 | 4 | esha .

. . . . . .

sūtraṁ ‖ guṇau . . . . . ka dhanaṁ ‖ 29a recto.

guṇ ābhyāso rūpa hīnaṁ labdhaṁ rū . . .

A 10. [27 *verso*] gives the answer of the problem given on fol. 29 *verso*, namely $x_1=9$, $x_2=7$, $x_3=10$, $x_4=8$, $x_5=11$, and the sums of the pairs are 16, 17, 18, 19, 20. (For general discussion see § 78, Part I.)

[27 *recto*.] Solution of a lost problem which may have been $xy-3x-4y\pm1=0$ of which solutions are: $x=\frac{3\cdot4-1}{-}+4=15$ $y=3+1=4$; $x=4+1=5$, $y=\frac{3\cdot4+1}{1}+3=16$. The quotations are from a *sūtra* very much like the one that follows.

The phrase *pṛithak rūpaṁ vinikshipya* 'having added unity in each case' appears to be a quotation from a lost *sūtra*.

[29*a*] is wrongly placed on plate XX. It should come directly under 27, for of the letters —*vaṁ sarvatra kārayet*, the top portions are on 27 *verso* and the bottom on 29*a recto*.

The writing is classed as α_2.

A 11.

i. bhyasa | .. / 1 | rū . . . . . chaturguṇaṁ paṁchaguṇaṁ hastagataṁ 1 recto.

dhanaṁ ja . . . . . paṁchaguṇaṁ 25 ||

navama sūtraṁ 9

ii. sūtraṁ || guṇau pṛithag rūpayutau yāchanā yukti samguṇāḥ ||

guṇanena guṇe . . rūpahīnena bhājitau |

viparīta yāchanā kshiptau guṇaśāster ayaṁ vidhiḥ ||

evaṁ sūtraṁ || dvitīya patre vivaritāsti ||

daśama sūtraṁ 10 ||

iii. sūtraṁ || aṁśāṁ viśoddhya chchhedebhya kuryātat parivartanaṁ ||

. . . sāsyaṁ tata projjhya dhanānviśa vinirdiśet ||

iv. udā° || paṁchānāṁ vaṇijā madhye *maṇi* vikrīyate kilaḥ

tatroktā maṇi vikrīta maṇi mūlyam kiyad bhavet

. . . . . dam

ardha tṛi-bhāga pādāṅśaṁ paṁcha-bhāga *śodamśa cha*

. . . . . †tato projjhyaḥ† sadṛiśaṁ kṛiyate . . . . jātā 1 verso.

| 120 | 90 | 80 | 75 | 72 tatra projjhya† jātaṁ 120 | 90 | 80 | 75 | 72
60 | 60 | 60 | 60 | 60

eshāṁ yoga kṛite jāta 437 ato . . . śesham 377 eśa maṇi mūlyaṁ |

A 11. [1 *recto.*] The position is uncertain. The 'find order' (33) places this leaf next to the fragments of folios 27, 29, 38, 39, 40. The writing is α_2 (there is a 'sickle' i). The numbered *sūtras* seem to place the leaf fairly early but they are not a very safe criterion. Note (ii) below seems to connect folios 1 and 27.

(i) Nothing intelligible. It ends the earliest numbered *sūtra* preserved.

(ii) I have not yet made out the meaning of this *sūtra*. Compare the opening phrase with the quotation on 27 *recto*. The metre is irregular. The reference to the second leaf is possibly to folio 27.

(iii) The *sūtra* means change $\frac{a}{b-}$ to $\frac{b}{b-a}$ and quotations from it are given on folios 1 *verso* and 2 *verso*.

(iv) The example is solved on 1 *verso* and 2 *recto* and appears to have been somewhat as follows:

The combined capitals of five merchants less one-half of that of the first, one-third that of the second, one-fourth that of the third, one-fifth that of the fourth, or one-sixth that of the fifth is equal to the cost of a jewel. Find the cost of the jewel and the capital of each merchant.

A 11. [1 *verso.*] This appears to give part of the solution and proofs of the question on 1 *recto*. Since $< \Sigma x - \frac{1}{2}x_1 = \Sigma x - \frac{1}{3}x_2 = \Sigma x - \frac{1}{4}x_3 = \Sigma x - \frac{1}{5}x_4 = \Sigma x - \frac{1}{6}x_5 = c$, we have $\frac{1}{2}x_1 = \frac{1}{3}x_2 = \frac{1}{4}x_3 = \frac{1}{5}x_4 = \frac{1}{6}x_5 = k >$ whence $\Sigma x = \frac{120+90+80+75+72}{60}k = \frac{437k}{60} = \frac{437c}{437-60}$.

If $k=60$ then $c=377$ and $x_1=120$, $x_2=90$, $x_3=80$, $x_4=75$, $x_5=72$.

Then follows a 'proof' which may be expressed by

$90+80+75+72=317$ and $317+\frac{1}{5}^{90}=377$
$120+80+75+72=347$ „ $347+\frac{1}{3}^{90}=377$
$120+90+75+72=357$ „ $357+\frac{1}{2}^{90}=377$
$120+90+80+72=362$ „ $362+\frac{1}{4}^{90}=377$
$< 120+90+80+75=365$ „ $365+\frac{1}{6}^{90}=377 >$

† Compare with *sutra* 11 on 1 *recto*.

A 11—*contd.*

chaturthāṁ saṅka sarvasvam ||

prathamasya saṅka ardham . . . 90 | 80 | 75 | 72 chaturnāṁ yoga 317 prathamārdheṇa sashṭibhir yutaṁ 377 eśa *prathamasya dhanaṁ* prathama dhanaṁ | tṛitīya chaturtha paṁchamasya dhanaṁ sarvasvaṁ 347 dvitīyā tṛi-bhāgaṁ 30 *eśa* yutaṁ 377 esa dvitīyasya dhanaṁ bhavati ||

puna prathama dvitīya chaturtha paṁchama . . sarvasvaṁ 357 tṛitīyasya pādam 20 eśa yutaṁ 377 esha tṛitīyasya *dhanaṁ bhavati* ||

punar api prathama dvitīya tṛitīya paṁchamasya 362 *chaturthasya pamcha-bhāga* 15 *eśa yutaṁ* 377 esha *chaturthasya dhanaṁ* bhavati || . . . .

A 12.

1. Sya dhanaṁ bhavati ||

atha pratha . . . . . tyaṁśashṭi śeshaṁ 377 || 2 recto.

atha dvitīyasya 120 30 80 75 72 evaṁ 377 dvitīyasya bhavati ||

atha tṛitīyasya kṛiyate 120 90 20 75 72 *evaṁ* 377 tṛitīvasya dhanaṁ bhavati ||

chaturthasya kṛiyate 120 90 80 15 72 evaṁ 377 chaturthasya dhanaṁ bhavati ||

paṁchamasya kṛiyate | sthāpanaṁ 120 90 80 75 12 | evaṁ paṁchamasya 377 |

A 12. [2 *recto.*] 1. This appears to be another 'verification' of the example on 1 *recto et verso*; and means

$<1\frac{1}{2}^{0}+90+80+75+72=377>$

$120+\frac{1}{2}^{0}+80+75+72=377$

$120+90+\frac{1}{2}^{0}+75+72=377$

$120+90+80+\frac{1}{2}^{0}+72=377$

$120+90+80+75+\frac{1}{2}^{0}=377$ and 'this is the measure of the price of the jewel.'

A 12—*contd.*

esha maṇi mūlyaṁ pra . . . . .

ll. udā° || anyonya vidita vibhavaṁ . . . . vaṇik*dvaya*ṁ |

tṛi . . . . . dalaṁ tatha . .

7+	3+	5+
12	12	6
12	12	6

2 verso.

†aṁśāṁ viśoddhya† visodhayet riṇaṁ sthitaṁ | esha . . . kṛiyate

19	7	11
12	4	6

†kuryātat parivartanaṁ†

12	4	6
19	7	11

chchhede

. . . . jātaṁ asya

924	836	798
1463	1463	1463

projjhya jātā

924	836	798

eshāṁ yutiṁ kṛiyate . . . jātā | 2558 | chchheda projjhyaṁ 1095 etan maṇi mūlyaṁ

A 12. ll. This is possibly the question solved on 2 *verso*.

[2 *verso*.] The general meaning is: since $x_2+x_3-(\frac{1}{2}+\frac{1}{3})x_1=x_1+x_3-(\frac{1}{2}+\frac{1}{3})x_2=x_2+x_1-(\frac{1}{2}+\frac{1}{3})x_3=c$, or $\Sigma x-(1+\ )x_1=\Sigma x-(1+\frac{1}{3})x_2=\Sigma x-(1+\frac{1}{3})x_3=c$, whence $\frac{1}{3}x_1=\frac{1}{4}x_2=\frac{1}{5}x_3=\Sigma x-c$ and $\frac{\Sigma x}{\Sigma x-c}=\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{924+836+798}{1463}$. Setting $\Sigma x-c=1463$ we have $x_1=924$, $x_2=836$, $x_3=798$; $\Sigma x=2558$ and $c=1095$ 'which is the price of the jewel.'

I do not, however, understand the form of the first statement; but see fol. 65 *verso* where $\frac{890}{40\ 42}$ means $\frac{890+40}{42}$.

† *aṁśāṁ viśoddhya* and *kuryātat parivartanaṁ* are quotations from *sūtra* 11 on fol. 1 *recto*.

A 13.

udā° || . . . . . dvitīyasya hayān navaḥ 3 verso.

ūshṭrā dasa tṛitīyasya . . . . . .

. . . . . pradattaṁ cha parasparaṁ

pṛithag dhanaṁ tu vaṇijām mūlyaṁ vā prāṇinām pṛithak

yadi . . . . vaktum tato me chchhindhi saṁsayaḥ ||

A 13. [3 *verso*.] Writing α_4. Note the looped medial *a* in the penultimate line. Possibly a double leaf. 'Find order' 49. The position is determined only by the writing and the numbered *sūtras* on the reverse. *Example*: One possesses 7 horses (α°=*aśva*), another 9 horses (*ha*°=*haya*) and a third 10 camels (*ū*°=*ūshṭra*). Each gives one of his animals to both the others (and then their possessions are of equal value). It is required to find the capital of each merchant or the price of each animal. If thou art able, solve me this riddle.

We have $(7-2)x_1+x_2+x_3=(9-2)x_2+x_3+x_1=(10-2)x_3+x_1+x_2=c$ or $\Sigma x-(7-3)x_1=\Sigma x-(9-3)x_2=\Sigma x-(10-3)x_3$, whence $4x_1=6x_2=7x_3=k$ and $\Sigma x=\frac{42+28+24}{168}k$. If $k=168$ then $x_1=\frac{168}{4}=42$, $x_2=\frac{168}{6}=28$, $x_3=\frac{168}{7}=24$. Also $7x_1=294$, $9x_2=252$, $10x_3=342$ are the original capitals, and $c=262$.

Mahāvīra gives the following example.

Rule.—The number of gems to be given away is multiplied by the total number of men. This product is subtracted from the number for sale: the continued product of the remainders gives rise to the value of the jewel provided the remainder relating to it is given up.

Example.—The first man had 6 sapphires, the second had 7 emeralds and the third 8 diamonds. Each by giving to each the value of a single stone became equal (in wealth to the others). *Answer* 20, 15, 12.

A 13—*contd.*

7	a°	9	ha°	ū°	10
1		1			1

vaṇijjakā 3 deyaṁ . . . . vaṇik piṇḍa hataṁ | piṇḍa 7 | 9 | 10 | deyaṁ 3 śuddha śesham 4 | 6 | 7 tata śeshaṁ paraspara kṛitaṁ guṇita jātaṁ | 168 | 168 | 168 | svaśeshena tu vibhaktaṁ

168	168	168
4	6	7

labdhaṁ 42 | 28 | 24 | esha pratyaika mūlyaṁ ekaikasya . . . . guṇitā jātāni asvai hayai ushṭrebhyaḥ 294 | 252 | 240 ekaikasya . . . . jātā 262 | 262 | 262 | etes sama dhanā

i. datvā ssamadhanā jātā prasta mūlyaṁ tad uchyatāṁ 3 recto.

4	ya°	5	go°	6	sa°
1		1		1	

evaṁ prasta mūlyaṁ 2 | 3 | 6 dattais samadhanā jātā 17 | 17 | 17

. . . .

trayodasama sūtraṁ 13

ii. sūtraṁ || ekayutānāṁ saṁkhyā dvi . . . . hīnā cha ||

evaṁ tāvat kāryaṁ yāvat purushai samā bhavati ||

saptama patre bhilikhita sthita

chatur-daśama sūtraṁ 14

iii. sūtraṁ || gatisyaiva viśeshaṁ cha vibhaktaṁ pūrva gaṁtunāḥ

tenaiva kālaṁ bhavati stha . . . . kena tu ||

iv. udā° || addhyardha yojana gate śata . . . .

. . . . . .

A 13. [3 *recto.*] This is the reverse because *sūtra* 15 obviously begins a new section (B).

i. This appears to be a companion example to that on 3 *verso*. The abbreviations are possibly *ya°* for *yava* 'barley,' *go°* for *godhūma* 'wheat,' *sā* for *sāli* 'rice.' Here (4—2) $x_1+x_2+x_3=(5-2)\ x_2+x_3+x_1=(6-2)\ x_1+x_1+x_2=c$ whence $x_1=2x_2=3x_3$ and $x_1=6$, $x_2=3$, $x_3=2$ and $c=17$.

ii. Not understood. The reference to the seventh leaf is now only tantalising. No recognisable quotations from the *sūtra* are preserved. The phrase *tāvat* ..... *yāvat* 'so much ..... as much' does not recur anywhere. In Bhāskara *tāvat* and *yāvat* (*tā°* and *yā°*) are used as algebraic quantities.

iii. The rule means $t=\frac{r_1 D}{r_2-r_1}$ where r_1 and r_2 are rates of progress and D is a given time. (See § 83, Part I.) The rule is quoted on 4 *recto* where *gatisyaiva viśeshaṁ cha* and *pūrva gaṁ* occur.

B 1.

i. sūtram ‖ dviguṇaṁ prabhavaṁ śuddhā dviguṇaṁ *niyatham tathā* 8 recto.

uttareṇa bhajech chheshaṁ labdhaṁ rūpaṁ vinirdiśet ‖

ii. udā° ‖ var*tate* bhṛitakaχ kaschi tatraiko dasha māśakaṁ |

pratyahaṁ kurute tatra karmaṁ bhaṭṭika mānavaḥ

dvitīyaṁ kriyate karmaṁ dvyādi tṛitayar uttaraṁ |

padaṁ tatra tu bhavati kena kālena sāsyatāṁ ‖

| ā° 2 / 1 | u° 3 / 1 | pa° 0 / 1 | prati° 10 / 1 |

‡dviguṇaṁ prabhavaṁ śuddhā‡ prabhavaṁ | 2 | dviguṇaṁ | 4 | niyata puna dvi

. . . . . | 16 | [uttarārdheṇa bhājayet] uttaram . . . . .

i. sūtram ‖ *hayor vibhajya gantavyaṁ ato bhāga* . gantata 8 verso.

ekaś cha gamana jñeya yutās samguṇya . . .

udā° ‖ niyo rathośvair daśabhir yujyate haya pamchakaṁ

gaṁtavyaṁ yojana *śatam* . . . . kim udbhavet

| ha 10 / 1 | haya lagna rathasya 5 / 1 | gantavyo yojana 100 / 1 |

†ĥayor vibhajya gantavyaṁ† tatra havā | 10 | gantavyaṁ yo° | 100 | †ato

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B 1. [8 *recto.*] The position of folios 8, 9 and 7 is very doubtful. They fit in nowhere perfectly. Their find orders are 48, 43 and 45; but 7 *recto* indicates that this find order is not of much value here. See the notes on fol. 7 *verso*. The writing is α4.

i. The rule is another variation of that given on 7 *verso* and means $t=\frac{2A-2a}{d}+1$ where A is a fixed rate and t $A=((t-1)\frac{d}{2}+a)t$.

ii. The example is $A=10$, $a=2$, $d=3$ whence $t=\frac{2\cdot10-2\cdot2}{3}+1$ $=6\frac{1}{3}$ and $s=\frac{1\cdot10}{6\frac{1}{3}}=63\frac{1}{3}$.

The phrase *dviguṇaṁ prabhavaṁ śuddhā* is quoted from the *sūtra* above; while the phrase *uttarārdheṇa bhājayet* was wrongly quoted and was afterwards cancelled: Compare with the *uttarārdheṇa bhājitam* quoted on 7 *verso*.

B 1. [8 *verso.*] i. The *sūtra* is partially reconstructed from the quotations in the solution below.

ii. The example is: There are ten horses of which five are yoked at a time to the chariot. How many changes should there be in a journey of one hundred yojanas and how much will each horse do?

The solution is $\frac{10}{1}\frac{0}{8}=10$ stages and $10\ \ 5=50$

Proof. 5×100 . . . .

Mahāvīra gives a similar example (vi, 158).

ravi-ratha turagāḥ sapta hi chatvāro'vā vahanti dhūryuktāḥ |
yojana-savitati-gatyaṁ ko vṛiddhaṁ ko chaturyoge ||

'It is well known that the horses of the Sun's chariot are seven. Four horses are yoked at a time. They have to perform a journey of 70 *yojanas*. How many times are they unyoked and how many times yoked'

Mahāvīra's solution is expressed thus:

The number of the total *yojanas* divided by the total number of horses gives the *yojanas* in turn. These *yojanas* multiplied by the optionally chosen number of horses to be yoked gives the measure of the distance to be travelled over by each horse.

That is $\frac{7}{7}=10$ is the length of each stage, and $10\times4=40$ gives the distance each horse works.

The solution is rather cryptic, but the interesting point is that the problem was a traditional one. Probably something of its original quality has been lost.

B 1—*contd.*

bhāga† *hṛite labdha* 10 tatra yuktāśva 5 etais saṁguṇya pariyoga jātaṁ

. . . . **yojanānyaikośva** rūḍha | pratyayaḥ paṁchabhis śata saṁguṇya

jātaṁ . . . *kṛi*yate || yadi da . . yojana paṁcha . .

B 2.

udā° || . . . . . tat samāptaṁ dvijanmabhi | 9 recto.

tat punas te samaṁ bhaktvā daśa . . samāptavān |

saṁkhyāya χ kati māchakshu kati viprā χ kati prashṭaṁ ||

ā° 1	u° 1	pa° 0	labdhaṁ 10
1	1	1	1

karaṇaṁ || ‡labdhaṁ dviguṇitaṁ kritvā‡ tatra labdhaṁ 10 | dviguṇaṁ

20	tathādvyūnaṁ	18	‡uttareṇa vibhājitaṁ‡	atrottaraṁ	1	anena
		1				

bhaktvā jātaṁ tad e*sha rūpādhi*kaṁ | 19 | ayaṁ praśhṇā brāhmaṇā ekona-
1

viṁśati

sthāpa . . .	ā° 1	u° 1	pa° 19	rūpoṇā karaṇena phalaṁ	190
	1	1	1		1

. . . . . 9 verso.

. . yo° 6	śa° yo° 1	yo° 70	gantavyaṁ
1	1	1	

B 2. [9 *recto.*] See the notes on fol. 7 *verso*. The writing is of the same style, α4. Possibly there are two leaves stuck together.

The example is a=1, d=1, A=10, and $10t=((t-1)\frac{1}{2}+1)t$ whence $t=\frac{21}{2}+1=19$ and by the *rūpona* method a=190.

Dr. Hoernle gave the following restoration:

"For a certain feast one Brāhman is invited on the first day, and on every succeeding day one more Brāhman is invited. For another feast 10 Brāhmans are invited on every day. In how many days will their numbers be equal; and how many Brāhmans were invited."

The use of the term *labdhaṁ dviguṇitaṁ kritvā, tathādyūnaṁ, uttareṇa vibhājitaṁ* and *rūpādhikaṁ* are probably quotations from a *sūtra*.

B 2. [9 *verso.*] The example probably meant: 'A and B start for a place 70 *yojanas* distant. A travelled at the rate of 1 *yojana* a day and B at the rate of 6. At what point on his return journey did B meet A?'

Since $\frac{x}{1}=\frac{140-x}{6}$, where x is the distance traversed by A, we have $x=\frac{140}{6+1}=20$ as given in the text, and since A travels at the rate of one *yojana* a day, this is also the time.

Proof by the 'rule of three' 1 day : 6 yo° :: 20 days : 120 yo°, and 70−20=50 and 70+50=120. Also 1 day : 170° :: 20 days : 20 yo°.

The abbreviation *sa°* may be for *sanairga* 'slow goer.'

B 2—*contd.*

a(la)bdhe saṁyoga | 7 / 1 | vibhaktaṁ | 1 / 7 | *ganta*vyena guṇitā jātān labdha

| 10 | dviguṇaṁ | 20 | eshālpasyaḥ ||

atha . . . ayam kālo jñeyaḥ anena kālenash shat yojanāni gantavyaṁ |

. . . bhyām ekayojanikasya samāgamo bhavati ||

tadyathā trai-rāśikena *pratya*ya | yady ekasya shaṭ yojanā tadā viṁśānāṁ kiṁ

| 1 / 1 | 6 | yo° | 20 / 1 | pha° | 120 / 1 |

atha saptati śoddhya śesha atra ssaptati | 70 | āgata paṁchāśa | 50 | *adhve*

. . . .

| 1 / 1 | di° | 1 / 1 | yo° | 20 / 1 | di° | pha° | yo° | 20 / 1 |

B 3.

i. . . . . 7 verso.

| ā° | 3 / 1 | u° | 4 / 1 | pa° | 0 / 1 | nitya datta | 7 / 1 |

†ādiṁ viśoddhya† . . . . *ādi* | 3 | niyataṁ | 7 | viśoddhya | 4 |

†uttarārdhena bhājitaṁ† | uttaraṁ 4 | *anena bhājita*ṁ 4 / 2 | jātaṁ | 2 |

†labdhaṁ sarupa† | esha rūpādhikaṁ | 3 | eśa kāla . . . . . .

| ā° | 3 / 1 | u° | 4 / 1 | pa° | 3 / 1 | rūpoṇa karaṇena phalaṁ rū° | 21 ||

dvitīyasya trai-rāśikena . . . | 1 / 1 | di° | 7 / 1 | 3 / 1 | di° | pha° rū° | 21

B 3. [7 *verso.*] Folio 7 is a very interesting sheet. The writing may be classed as α4. On examining the original I noted that it was a double sheet, but the reproduction (Plate vi) might lead one to conclude that it was a palimpsest. Probably, however, the writing underneath is showing through, or the faint writing marks have been impressed from the contiguous leaf. The two sides are definitely disconnected by their contents and the right side has now been definitely located between folios 6 and 65. Folios 7 (*verso*), 8 and 9 are difficult to place. Indeed there seems to be some duplication. Folio 5 certainly follows folio 4 and section C cannot very well include folios 7 (*verso*), 8 and 9.

i. The problem is $7t=((t-1)\frac{1}{4}+3)t$ whence $t=\frac{2(7-3)}{4}+1=3$. By the *rūpoṇa* method $s=[(3-1)\frac{1}{4}+3]t=21$ and by the 'rule-of-three' 1 : 7 : : 3 : 21.

The phrases *ādiṁ viśoddhya, uttarārdhena bhājitaṁ* and *labdhaṁ sa rūpa* are quotations from a lost *sūtra*. Compare with fol. 8 *recto*.[1]

B 3—*contd.*

esha ssamadhanā jātā ||

ii. udā° || ādyeka uttara dvayaṁ dvitīya paṁcha pratyahaṁ |

kena kālena samatāṁ vada me gaṇakottama ||

ā° 1	u° 2	pa° 0	niyata nityaṁ 5
1	1	1	1

†ādiṁ viśoddhyā† . . . . . .

B 3. ii. The problem is $5t=((t-1)\frac{1}{2}+1)t$ whence $t=<2(\frac{5}{2}-1)+1=5$ and $s=25$ >.

B 4.

i. . . . . yojana paṁchakaṁ | sapta dināni [4 recto.]

tasyaiva gatasya | parata dvitīya nava yojanaika gatake . . . taṁ

1	di° 5	yo°	dina 7	gatasya	gata yojana 35	dvi° 1	di° 9	yo°
1	1		1		1	1	1	

†gatisyaiva viseshaṁ cha† . . . yate | gati 5 | 9 | viśesha ṁ | 4 |

vibhaktaṁ 1/4 pūrva gata 35 esha pāder guṇitaṁ | 35/4 . . bhir

dinai sama gatī bhavanti nava yojanaṁ ||

pratyaya trai-rāśikena

1	di°	5	yo°	35	. . .
1		1		4	
1	di°	9	yo°	35	pha° . . .
1		1		4	

ii. udā° || ashṭā-daśa yojanā ekena dine yātī |

tasyāshṭa dinā gatasya |

dvitīya paṁcha-viṁśe yojanā dine yāti |

B 4. [4 *recto.*] The writing changes, due possibly to the use of a different pen, but it is different and may be termed x3. This leaf is closely connected with fol. 3 *recto* and with fol. 5.

i. The example may be restored: One goes at the rate of 5 *yojanas* for 7 days and then a second starts at the rate of 9 *yojanas* a day. When will they have traversed equal distances?

The phrase *gatisyaiva viseshaṁ cha* is a quotation from *sūtra* 15 (fol. 3 *recto*) and *pūrva gata* is a reference to the same rule.

The solution is $t=\frac{7\cdot5}{9-5}=\frac{35}{4}$ days. 'Proof by the rule of three' $1:5::\frac{35}{4}:\frac{175}{4}$ and $1:9::\frac{35}{4}:\frac{315}{4}$ <and $\frac{315}{4}-\frac{175}{4}=35$>.

One travels at the rate of 18 *yojanas* in one day for a period of 8 days. A second goes at the rate of 25 *yojanas* in one day. Determine in what time.

The eleventh leaf must have been close by: indeed *pūrvepi* seems to indicate that it was 'just before.

B 4—*contd.*

kena kālena sāsyatāṁ ||

evaṁ ekā-daśama pattre bhilikhita purvepi ||

paṁcha-daśama sūtraṁ 15

iii. sūtraṁ || *ādy*or viśes[h]a kartavyaṁ uttarasya viseshataḥ

vibhaktaṁ muttare . . . . . .

4 . . . . . 4 verso.

uttaraṁ 2 1 vibhaktaṁ 1 2 ādi śesha 2 1 jātā 1 1 dviguṇaṁ

2 rūpa saṁyutaṁ 3 esha . . . . saṁkalite

pratyaya | *padaṁ* . īṇā ubhaye sthāpitavyā rūpoṇā karaṇe phalaṁ 21 . .
21 dvi

kiṁ prabhūtepi likhite || shodaśama sūtraṁ 17 sūtre bhrāntim asti

ii. sūtraṁ || ādyor viśesha dviguṇaṁ chaya suddhir vibhājitaṁ |

rūpād*hikam tathā* kāla*ṁ* gati sāsyaṁ tadā bhavet ||

iii. udā° || dvayādi tri chayaś chaiva dvitiya tryādi-k-ottaraḥ

dvayo cha bhavate paṁthā kena kālena sāsyatām ||

sthāpanaṁ kṛiyate ||

ā° 2 1	u° 3 1	pa° 0 1	dvi°	ā° 3 1	u° 2 1	pa° 0 1

karaṇam |‡ ādyor viśesha‡ . . . . . .

B 4. iii. The rule means (?) $t=2^{\frac{(a_1-a_2)}{d_2-d_1}}+1$. Note that the next *sūtra*, on the reverse, commences with the same phrase *ādyor viśesha*.

[4 *verso*.] i. The example was $a_1=4$, $d_1=3$; $a_2=6$, $d_2=1$. Where a_1 and a_2 are the first terms and d_1 and d_2 are the increments of arithmetical progressions, the sums of which were equal. Therefore $(t-1)\frac{3}{2}+4=(t-1)\frac{1}{2}+6$ whence $t=2^{\frac{(6-4)}{3-1}}+1=3$.

The proof is by the *rūpona* method, namely, $a_1=((3-1)\frac{3}{2}+4)3=21$ and $a_2=((3-1)\frac{1}{2}+6)3=21$. But 'why should it be written out in full?' See Part I, § 73.

The remark that the *sūtra* is wrongly numbered was probably added later by some one other than the original scribe. The next *sūtra* is numbered 18 (fol. 5) and so on. This is not a copyist's error: it is one of an original MS.

ii. The rule is much the same as the previous one and means that $t=2^{\frac{(a_1-a_2)}{d_2-d_1}}+1$ when $((t-1)\frac{d_1}{2}+a_1)t=((t-1)\frac{d_2}{2}+a_2)t$. The rule is quoted below and on fol. 5 *recto*.

iii. The example gives $a_1=2$, $d_1=3$; $a_2=3$, $d_2=2$ < whence $t=3$ and $s=15$ >.

C 1.

ā°	5	u°	6	pa°	0	dha°	0		5 recto.
	1		1		1		1		
ā°	10	u	3	pa	0	dha°	0		
	1		1		1		1		

karaṇaṁ | ‡adyor viśeshaṁ‡ | ādi . . . . . ‡chaya śuddhi‡ chayaṁ

6 | 3 | śuddhi 3 / 1 ādi śesha 5 dviguṇaṁ 10 uttara viśesha 3

vibhaktaṁ | 10 / 3 | sa rūpaṁ 13 / 3 aneṇa ka . . . samadhana bhavanti |

pratyayaṁ | rūponā karaṇena phalaṁ 65 ; dvi 65 ; esha padaṁ . . . .

ashṭādaśama sūtraṁ 18

ii. (sūtram) || dina gaṇanaṁ ādi rahitaṁ *dvi*guṇaṁ tachchottareṇa saṁyutaṁ |
pratinihita ātmagaṇaṁ *jaṇam kshepa saṁjñako* rāśi |
ashṭottara guṇite kshepa saṁjñako datvā mūlaṁ
pra*tinihita yutam dviguṇottara bhājitam*

. . . . . hataṁ 30 . . . †dina gama- 5 verso.

ṇam ādirahitam‡ dina gaṇana yojana pañcha 5 adi 3 rahitaṁ

jātaṁ 2 †dviguṇaṁ† 4 ‡tachchottareṇa saṁyutaṁ† 8 . . .

†ātmaguṇaṁ† 64 ; esa †kshepa saṁjñako raśi | ashṭottara saṁgu . . .

labdha rāshi | 30 ashṭa guṇaṁ 240 ; uttareṇa guṇaṁ uttaraṁ 4 |

guṇitaṁ jātaṁ 960 ; †kshepa saṁjñako datvā† | tatra kshepa saṁjñ . . .

64 ; yutaṁ jātaṁ 1024 asya mulaṁ 32 ; †pratinihita† . . . .

8 yutaṁ jātaṁ 40 u . . . . . .

C 1. [5 *recto.*] The writing is the same as that on folio 4, namely 23, but it changes again in the middle of 5 *recto*.

i. The example is $a_1=5$, $d_1=6$; $a_2=10$, $d_2=3$, where $(t-1)^2_{\frac{1}{2}}+5)t$ & $((t-1)+10)t$, and the solution is $t=2(a_2-a_1)(d_1-d_2)+1$ or $2(10-5)(6-3)+1=\frac{1}{2}^3$.

Proof by the *rūpona* method $s_1=<((\frac{3}{2}^2-1)^2_{\frac{1}{2}}+5)^3_{\frac{1}{2}}-((\frac{3}{2}^2-1)^2_{\frac{1}{2}}+10)^3_{\frac{1}{2}}>=65$.

The *sūtra* number should probably be 17. See fol. 4 *verso*.

ii. The writing now changes to what may be termed the α4 style. The rule means that < if $DT+Dt=((t-1)\frac{3}{2}+a)t>$ then

$$t=\frac{(d-2(a-D))^2+8dDT}{2d}+d-2(a-D)$$

where D and T are fixed quantities and a, d and t are elements of an arithmetical progression of which a and d only are given. The quantity designated *pratinihita* 'set aside' is $d-2(a-D)$, while the *kshepa saṁjñako rāśi* 'the quantity known as *kshepa*' is $\{d-2(a-D)\}^2$.

[5 *verso.*] Writing α4. Notice a semi-looped medial *e* near the end

i. The example is < D=5, T=6, a=3, d=4; hence $t=\frac{\sqrt{(2(5-3)+4)^2+8.4.5.6}+2(5-3)+4}{2.4}>$. The solution proceeds step by step thus: DT=5.6=30, D−a=5−3=2, 2(D−a)=4, 3(D−a)+d=4+4=8; (2(D−a)+d)²=64 and 'this is known as the *kshepa* quantity'; 8DT=240, 8DTd=960; 8DTd+(2(D−a)+d)²=1024, $\sqrt{1024}=32$; 2(D−a)+d+32=40; and < $\frac{40}{8}=5$>.

Almost the whole of the *sūtra* on 5 *recto* is quoted here and on the following pages.

27481

C 2.

i. śike pratyayaṁ | 1 1 | 5 1 | 5 1 | phalaṁ . . . . anenas saha 55 eśa samābdhānaṁ ‖ 6 recto.

ii. udā° ‖ ādi paṁchaṁ uttaraṁ *trīni* naro yojana gamyate |
dvitīya pratidinaṁs sapta gatasya dina paṁchakaṁ |
kena kālena samatām katthyataṁ gaṇakottama ‖

| ā° 5 1 | u° 3 1 | pa° 0 1 | prati° gati 7 1 | dina 5 1 |

paṁcha dina ga . . . yojanikaṁ yojana | 35 |
karaṇaṁ | †dina gamanam adi rahitaṁ† tatra dina gamanaṁ | 7 |
†ādi rahitaṁ† ādi 5 rahitaṁ . . . . . .

i. . . . anena guṇitaṁ jātaṁ | 840 | †samjñako datvā† tatra kshepa rāshi | 49 | 6 verso.
datvā jātaṁ 889 | . . . . dāna dadāti samaṁ | karaṇī kriyate

ii. sūtraṁ ‖ akṛite sli*shṭha kṛityūnā śesha* chchhedo dvi-saṁguṇaḥ
tad vargaḥ dala saṁslishṭha hṛiti *śuddhi kriti kshaya*ḥ
anena sutreṇa slishṭha mūlaṁ ānaya svamatimā . . . . .

i. . . labdhaṁ mūlam | 29 48 58 | †pratinihitaṁ† | 7 | anena yutaṁ | 36 48 58 |

. . . | 2136 58 | †dviguṇottara bhājitaṁ | | tato . . . . . .

C 2. [6 *recto.*] i. Continues the example. 'Proof by the rule of three' 1 : D :: t : Dt or 1 : 5 :: 5 : 25 and DT+Dt=30+25=55.

ii. The next example is D=7, T=5, a=5, d=3; hence $t=\frac{\sqrt{(2(7-5)-3)^2 \cdot 4 \cdot 7 \cdot 5}+2(7-5)+3}{2 \cdot 7}$

Part of the solution is lost < DT=35, 2(D−a) | d=7, 7²=49 >. It is continued on 6 *verso*.

[6 *verso.*] i. Continues the solution: 8DTd=840; 8DTd+(2(D−a)d)²=889. Here the solution breaks off in order to tackle the problem of obtaining the root of a surd quantity, and a subsidiary (un-numbered) *sutra* is given.

ii. The rule recurs on folios 56 *recto* and on 57 *verso*, and with the help of these other versions it has been restored as above.

The rule means that an approximate root of $\sqrt{A^2+b}$ is $A+\frac{b}{2A}$ and that the difference between the squares of these two quantities is $(\frac{b}{2A})^2$; and that by continuing the process closer approximations can be obtained. For a discussion of this rule see Part I, §§ 68, 69, 85. The three versions as they now stand are –

akṛite śli . . . . . chchhedo dvi-saṁguṇaḥ 6 Verso.
tad vargaḥ dala | saṁślishtha | hṛit . . . . yaḥ
. . . kṛity ūnān sesha chchhedo dvi-saṁguṇaṁ | 56 Recto.
tad varga . . . . ślishthaḥ hṛiti śuddhi kṛiti kshayaḥ ‖
akṛite ślishṭha kṛity ūnā śesha chchhedo dvi . . . . 57 Verso.
varga dala saṁślishtha hṛiti śuddhi kṛiti kshayaḥ . . . .

iii. The solution is resumed: < since 889=841+48=29²+48 > the first approximation to $\sqrt{889}$ is $29\frac{48}{58}$ and (terming this q_0) we have $q_1+2(D-a)+d=29\frac{48}{58}+7=36\frac{48}{58}=\frac{2136}{58}$ < and $t_1=\frac{2136}{58\times 6}=\frac{1}{6}\frac{1}{8}$ > where t_1 is an approximate value depending on q_1.

C 3.

6		447	dalitā	447	. . .	sāsye yutaṁ	737	pada	7 recto.
1		29		58			58		
8									
60*		ghnā	tatra padaṁ	178	anena guṇitaṁ jātaṁ		65593		
16	cha°			29			841		
60*									
33	li°	śli	tya śeshaṁ kṛiyate	65569	bhage hṛite				
60*				841					
6	vi°								
60*									
śe°	6	pratyayaṁ trai-rāśikena		1	7 yo	178	phalaṁ		
29				1	1	29			

yojana 42 śe 28 niyataṁ tena . . . 77 .
29

ekona-viṁśatima sūtraṁ 19 ||

C 3. [7 *recto.*] This continues the example started on fol. 6 *recto.* [The numbers marked with asterisks are change-ratios (see Part I, §§ 103–105).] The set of figures on the left expresses $\frac{1}{q_0}$ as a sexagesimal fraction (see Part I, § 58), i.e., $\frac{1}{29} = 6 + 8' + 16'' + 33''' + 6\,\frac{6}{29}''''$. The portion of the statement above the 16 is missing but the restoration is certain. Of the abbreviations *cha°* has not yet been identified; *li°* stands for *lipta* (Gk. λεπτόν), *vi°* for *vilipta*, *śe°* for *śeshaṁ* 'remainder.' In Hindu astronomical works *lipta* means a 'minute of arc,' and *vilipta* 'a second of arc.' This use of the sexagesimal notation for arithmetical purposes in an Indian work is unique. The solution proceeds to find the approximate value of s_1 which depends on t_1 and ultimately q_1. We have $s_1=((t_1-1)\frac{d}{2}+a)t_1$. Now $(t_1-1)d=(\frac{1}{29}-1)3=\frac{1}{9}\frac{4}{1}$, $(t_1-1)\frac{d}{2}=\frac{1}{9}\frac{4}{1}$; $(t_1-1)\frac{d}{2}+a=\frac{1}{9}\frac{4}{1}+5\cdot\frac{1}{9}\frac{4}{1}$; and $((t_1-1)\frac{d}{2}+a)t_1=\frac{1}{9}\frac{9}{1}\frac{6}{1}$. $\frac{1}{9}\frac{9}{1}\frac{6}{1}=\frac{6}{8}\frac{6}{4}\frac{6}{1}\frac{9}{1}$.

But $DT+Dt_1-7(5+\frac{1}{29}\frac{7}{1})=\frac{6}{8}\frac{8}{4}\frac{6}{1}\frac{9}{1}=77\frac{1}{29}\frac{4}{1}$.

'Proof by the rule of three': 1 . 7yo° $\frac{1}{29}\frac{7}{1}$: $42\frac{29}{60}$ and ·· $48\frac{7}{29}+35=77\frac{1}{29}\frac{4}{1}$ ·.

[Note that $\frac{6}{8}\frac{8}{4}\frac{6}{1}\frac{9}{1}\cdot\frac{6}{8}\frac{8}{4}\frac{6}{1}\frac{9}{1}=\frac{24}{841}=\left(\frac{48}{2\cdot29}\right)^2>\frac{1}{841}<\frac{1}{841}$. This process of reconciliation is explained in Part I, § 85.]

The *sūtra* number should probably be 18. See fol. 4 *verso.*

C 4.

ā°	1	u°	1	pa°	0	60		65 verso.
	1		1		1	1		

karaṇaṁ || †ashṭottaraghne guṇite | | ashṭa ghanaṁ | 480 | uttara ghana . . . dvi-ghnam ādi | ādi dvi-guṇa | 2 | chayojjhitam | cha . . . . uttaraṁ | ato uttaraṁ pātayitvā ekaṁ bhavati | 1 | va . . . nikshipya dhanasya | 481 | mūlaṁ ślishṭha karaṇyā | 21 | . . . . .
40
42

vaṁśaṁ | 882 | śeshaṁ chatvāriṁśa pṛithak sthāpya | 40 . .
40
42

yojyaṁ 922 | tan mūla varjitaṁ || tan mūlaṁ 880
42

C 4. [65 *verso.*] Folio 65 consists of two leaves stuck together. The writing on both sides may be classed as α4. The left side has no direct connexion with fol. 7 *recto* but it belongs to the same section.

The *sūtra* here quoted from is lost, or hidden, for possibly when folios 7 and 65 are separated it may be discovered. It may be said to be one of the most important *sūtras* of the whole work judging by the care and elaboration with which it is illustrated. It must mean that < when $s=((t-1)\frac{d}{2}+a)t$ then $t=\frac{\sqrt{(2a-d)^2+8ds}-(2a-d)}{2d}$ > where a, d, t and s are respectively the first term, the common difference, the number of terms and the sum of an arithmetical progression.

The example is a=1, d=1, s=60; hence $t=\frac{\sqrt{(3-1)^2+8.1.60}-(2-1)}{2.1}=\frac{\sqrt{481}-1}{2}$.

The solution proceeds 8ds=480, 2a−d=1, $(2a-d)^2+8ds=481$; by the square-root method (see fol. 6 *verso*) the first approximation is $21\frac{40}{42}=\frac{882+40}{42}=\frac{922}{42}$ and < $t_1=(\frac{922}{42}-1)\div2=\frac{880}{84}$ >

C 5.

880	964	guṇita jātaṁ	848320	chatvāriṅśa pṛithak sthānāṁ vargaṁ 56 verso.
84	168		14112	

1600	esha uparā pātya śeshaṁ	846720	vartya jātaṁ	60
		14112		

. . . . . 21 *teshā*m varggah tasthāt . . . . . 56 recto.
20
21

akṛite śliṣhṭha kṛityūnān śesha chchhedo dvi-saṁguṇaṁ |

tad varga dala saṁśliṣhṭhaḥ hṛiti śuddhi kṛiti kshayaḥ ||

†śesha chchhedo dvi-saṁguṇa† kṛi . . . . .

21					21 bha
20	400	dala 1	saṁślishṭhaḥ		20
21	441	2			21+

śeshaṁ pātya . . . . . dvā bhājita . . . †adhaṁ upare uparaṁ† guṇitavyaṁ vargaṁ yāva marjaye

. . .	425042	400	śeshaṁ	424642
	19362	19362		19362

C 5. [56 *verso*.] Continues the example. $s_1 = ((t_1-1)\frac{1}{2}+1)\,t_1 = t_1\frac{(t_1+1)}{2} = \frac{840}{84}\cdot\frac{964}{168} = \frac{848320}{14112}$, but $< \frac{c_1}{a_1} = \left(\frac{40}{2.21}\right)^2/8. > = \frac{1600}{14112}$ and $\frac{848320-1600}{14112} = \frac{846720}{14112} = 60$.

The bottom half of fol. 56 *verso* is blank but the example is continued on 56 *recto*.

[56 *recto*.] This continues the example given on fol. 65 *verso*. The top part of the leaf is much broken up; but the square-root rule (see fol. 6 *verso*) is given. Why this rule is repeated is not quite understood nor is it understood why it comes between two approximations of the same surd. Anyhow the general aim is clear: since the first approximation is $21\frac{8}{17}$ the second is given by

$$q_7 = 21\tfrac{8}{17} - \tfrac{1}{2}\cdot\left(\tfrac{8}{17}\right)^2/21\tfrac{8}{17} = 21\tfrac{8}{17} - \tfrac{400}{441}\times\tfrac{21}{461} = \tfrac{424,642}{19362}$$

C 6.

405280	444004	. . . . . ardhaṁ kartavyaṁ . . . . . 64 recto.
38724	38724	

405280	444004	saṁguṇya jātaṁ | a . . . . hrarā hareshu guṇ
38724	77448	

. . .	179945941120	asya ūrdhaṁ . . . . . 160000+
	2999096352	

C 6. [64 *recto*.] and $t_2 = \left(\frac{424642}{19362}-1\right)+2 > \frac{405280}{38724}$. Also $s_2 = \frac{t_2(t_2+1)}{2} = \frac{405280}{38724}\cdot\frac{444004}{77448} < = \frac{179,945,941,120}{2,999,096,352} >$ and $s_2 - \frac{160,000}{2,999,096,352} = \frac{179,945,781,120}{2,999,096,352}$ $= 60$.

C 6—*contd.*

. . . †śesha chchhedo dvi-saṁguṇaṁ | 6 5 12 | śeshaṁ paṁchakaṁ pṛithak . 64 verso.

. . . ansās vaṁsaṁ | 77 12 | tan mūla varjitaṁ | tan mūlam . . . . .

dvi-guṇottara saṁbhaktaṁ | 65 24 | esha padaṁ || . . . . yanaṁ

| ā° 1 1 | u° 1 1 | pa° 65 24 | rūpoṇā | 41 24 | . . . . . dalita | 41 48 |

ādi saṁyutaṁ | 89 48 | . . . . . .

C 7.

. . . . . †ashtottara-ghne guṇite† 40 dvi-ghnam ādi cha . . 57 verso.

. . . . nikshipya | 41 | mūlaṁ | 6 5 6 | †śesha chchhedo dvi-saṁguṇa†

. . . . . śuddhaḥ tasmāt

akṛite ślishṭha kṛity ūnā śesha chchhedo dvi *saṁguṇaḥ* |

tad varga dala saṁślishthaḥ hṛiti śuddhi kṛiti kshayaḥ ||

†akṛite slishtha† . . . . . tada dvi-saṁguṇa kṛita

| 6 5 12 | tad vargatam 12 | 6 5 12 | 25 144 | dala . . . . . |

25 1848 | . . . 11833 1848 | hṛi | 1848 | kṛiti kshaya kritaṁ : eśa 57 recto.

mūlaṁ || tan mūlaṁ . . . . mūlaṁ ekaṁ 1 esha sadṛiśe pātita jāta

9985 1848 | . . . sambhaktaṁ uttaram dvi-guṇaṁ 2 anena bhaktvā | 9985 3696 |

esha paṁchakasya padaṁ || asya pra . . . . .

sūtraṁ || eko rāśi dvidhā sthāpyaś *chayase* . . . . .

O 6-7. 64 *verso* 57 *verso* and 57 *recto* are all (except the last line) concerned with one example the beginning of which is lost. The example is < a=1, d=1, s=5; therefore $t=\frac{\sqrt{4i-1}}{2}$>. The first approximation to $\sqrt{41}$ is $q_1=6\frac{5}{12}=\frac{77}{12}$ and $t_1=\frac{5}{12}$. Therefore $s_1=((\frac{5}{12}-1)\frac{1}{12}+1)\frac{5}{12}=(\frac{1}{12}\cdot\frac{5}{12}+1)\frac{5}{12}=\frac{5}{12}\cdot\frac{5}{12}=<\frac{5786}{11852}=5\frac{25}{11852}=5+\frac{1}{12}(\frac{5}{12})^3>$. The second approximation is introduced by the square root rule (as previously on fol. 56 *recto*) and is given by $q_2=6\frac{5}{12}-\frac{1}{12}(\frac{5}{12})^2/6\frac{5}{12}=\frac{11856-25}{1848}=\frac{11833}{1848}$ and $t_2=\frac{1}{12}(\frac{11833}{1848}-1)=\frac{1}{12}\cdot\frac{9985}{1848}=\frac{9985}{3696}$ and 'this is the number of terms when the sum is five.'

Apparently a proof followed introduced by a *sūtra* of which, unfortunately, only a fragment remains.

C 8.

i. | 10225 / 32800 | dalitā | 10225 / 65600 | . . . . . ādi yutaḥ | 108625 / 65600 | padaghnā 45 recto.

pada *saṁyutā* . | 6455040625 / 3227520000 | ato pañcha-viṅśa . . . . uparāḥ

| 6455040000 / 3227520000 | . . . . labdhaṁ 2 esha dhanaṁ ||

ii. | ā° 1 / 1 / 2 | u° 1 / 1 / 2 | padu 0 / 1 | dhanu 7000 / 1 |

. . . . . | 384 asya varga 147456 | akṛi . . 21743271936 45 verso.

esha sarva guṇitā karaṇi . . . . kṛitvā bhājita jātaḥ 1158+ / 671250 aṁśair

aṁśā guṇaye . . . . . raśi varjya jātaḥ

| 579 / 768 / 1158 | 294912 / 777307500 | 579 / 515520000 / 777307500 | 294912+ / 777307500 |

śeshaṁ | 579 / 515225088 / 777307500 | . . . . . | 450576267588 / 777307500 |

dvayena mūle . . . . . .

C 8. [45 *recto.*] i. The greater portion of this example is lost, but can be restored. The example was < a=1½, d=1½, s=2; whence $t = \frac{\sqrt{10}-3}{6}$. The first approximation to $\sqrt{10}$ is $q_1=10\frac{1}{2}$ and the second is $q_2=10\frac{1}{2}-\frac{1}{2}(\frac{1}{2})^2/10\frac{1}{2}=10\frac{81}{828}$. This gives $t_2=\frac{10\frac{81}{828}-3}{6}=\frac{59425}{49200}$, and $s_2=((\frac{59425}{49200}-1)1\frac{1}{2}+\frac{1}{2})\frac{59425}{49200}\cdot\ldots=(\frac{10225}{32800}\cdot\frac{1}{2}+\frac{1}{2})\frac{59425}{49200}=(\frac{10225}{65600}+\frac{1}{2})\frac{59425}{49200}=\frac{108625}{65600}\cdot\frac{59425}{49200}=\frac{6,455,040,625}{3,227,520,000}$. < Now $\frac{s_2}{sd}=\frac{625}{3,227,520,000}$ (see Part I § 86 (v)) > and $s=s_2-\frac{s_2}{sd}=\frac{6,455,040,625-625}{3,227,520,000}=2$.

ii. The statement without any formal question should be noted. The example is a=1½, d=1½, s=7000. The first part of the solution is lost but a good deal of the later working is preserved on folios 45 *verso* and 46 *recto*. We have $q_1=579\frac{584}{670}$. (See *part* I, § 86 (vi).)

45 *verso*. The second approximation is given by $q_2=579\frac{768}{1158}-\frac{1}{2}(\frac{584}{670})^2/579\frac{768}{1158}=579\frac{768}{1158}-(\frac{584}{670})^2\frac{1158}{671250}=579\frac{768}{1158}-\frac{294,912}{777,307,500}=579\frac{515,520,000-294,912}{777,307,500}=579\frac{515,225,088}{777,307,500}=579\frac{450,576,267,588}{777,307,500}$. Continued on folio 46 *recto*.

C 9.

448244345088 / 4663845000 | 443580500088 / 4663845000 | ------- 221790250044 / 1554615000 dalitā e . 46 recto.

110895125022 / 1554615000 ādi saṁyuta 113227047522 / 1554615000 pada-ghnā 5075338376274674327193 6 / 7250483394675000000

. . . . . karaṇi pāta | 21743271936 pātita jātā uparānyāsa sthāpa .

5075338376272500000000 0 / 7250483394675000000 bhā 7000

C 9. [46 *recto.*] Continued from 45 *verso.* $t_3=\frac{450,870,26,588}{777,207,500}-3)\div 6=\frac{448,244,345,088}{4,663,845,000}$ and $t_3-1=\frac{443,580,500,088}{4,663,845,000}$. $(t_3-1)\,d=\frac{221,790,250,044}{1,554,615,000}$, $(t_3-1)\frac{d}{2}=\frac{110,895,125,022}{1,554,615,000}$, $(t_3-1)\frac{d}{2}+a=\frac{113,227,047,522}{1,554,615,000}$ and finally $a_3=((t_3-1)\frac{3/2}{2}+\frac{a}{2})\,t_3=\frac{50,753,383,763,746,743,271,936}{7,250,483,394,675,000,000}$. New $\frac{a_3}{a_1}=\frac{31,743,271,936}{7,250,483,394,675,000,000}$ and $s=s_3-\frac{a_3}{a_1}=\frac{50,753,383,763,726,000,000,000}{7,250,483,394,675,000,000}=7000$.

D 1.

maḍe 8 | maḍe 6 | maḍe 3 . . . . kā 20 apara prashṭaḥ pārā 46 verso.

a i e vīhujaṇa vī ha . . hai . . . ṇa | gore jā ma cha | uppaṇe
sā male a . . ḍha pa . ḍhale āpot diṇe āgaṇe vīhujaṇa ehu vī . . . . .
karaṇaṁ | trai-gore varehahipaṇehi sā . . . . .

D 1. [46 *verso.*] Writing α 4. Find order 9. This is quite unintelligible to me.

D 2.

tola 5 . . . . . 70ª recto.

. . . . . 35/2 | ete bhāgā . . . . . 70ᵇ recto.

. . . . 17/5 | 117/70 . . . . . .

. . . . . 2 . 0 rītā 7 . . . . . pala 2 tola 1 . . . pala 6 || 70ᶜ recto.

udā° || samā . . . . . ṇapeśī kṛitāni cha
dvecha tisraś . . . . . .
tisra samādāya] tulitāni trayo-daśe |
. . . . . ekaikasya sārdhayaḥ . . .

| 1/2 1/3 1/4 d . . . . .

prakshepa yuktyā phalaṁ . . . . 70ª verso.

. . . . . ri . . . . . ri 70ᵇ verso.

gunya phala rāśi . . . . .

katram pala 8 70ᶜ verso.

D 2. [70]. Folio 70 consists of 5 scraps not obviously connected. The writing may be classed as α_p. The 'find order' is 65 and this and the five following fragmentary leaves are placed in their 'find order,' for want of some more reliable basis of classification.

70 *recto* is mostly unintelligible but $x(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})=13$ and $x=12$ is a solution.

70 *verso.* Here $x(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})=65$ and $x=60$ is obviously connected in some way with the example on 70ᶜ *recto* but they are two separate examples.

D 2—*contd.*

udā° ‖ **ardha tṛi . . . dāṅśā paṁcha śashṭi nṛipo dadau |**

sevakānāṁ tu dī . . . . . .

1	1	1	dṛishya	65	sadṛi	. . . . .
2	3	4		1		

. . . . . .

D 3.

2	2	2	dṛishya	. . . . .	ato sadṛisha	. . .	hakaṁ			69 recto.
5	6	7								

upari māṁsaṁ taṁḍulā bhavanti . . . . . chatvāliṁśa | dūnā chau

rāśi . . eta taṁḍulā | dvā-chatvāriṁ . . . . . vanti ete vṛihakā

sarvattraḥ sthāpanaṁ asya . . . . .

pratyaya trai-rāśikena	5	ā°	2	taṁ°	210	pha°	taṁ°	84	
	1		1		1				

. . ṭyasya kṛiyate |

	6	. . . . .							
	1								

. . . . . yate *rāśih*

7	2	210	phalaṁ						
1	1	1							

. . . . . katraṁ 105 . . 69 verso.

udā° ‖ . . . . tṛibhir dattai tṛiguṇā tṛiguṇena tu |

. . . . . tad uchyatāṁ ‖

1	3	9	dṛishya	130	prakshepa	10	30	90	ekatram	130	
1	1	1		1							

. . . vān ‖ taṁ śataṁ tṛibhir datyai paravaptṛā pavaptṛi kai . . .

4	6	9	dṛi° 190	. . . . .	40	60	90	ekatram	
1	1	1							

D 3. Folio 69 consists of four pieces but is not quite so shabby as folio 70, for the two larger pieces fit together.

[69 *recto*]. The statement means $x(\frac{1}{5}+\frac{1}{6}+\frac{1}{7})=214$ whence $x=210$. The 'proof by the rule of three' is

5 *ā°* : 2 *taṁ°* : : 210 : 84 *taṁ°* < and 84+70+60=214 >
6 *ā°* : 2 *taṁ°* : : 210 : 70 *taṁ°*
7 *ā°* : 2 *taṁ°* : : 210 : 60 *taṁ°*

[69 *verso*]. Here $x(1+3+9)=130$ whence $x=10$ and the numbers are 10+30+90=130. Again $x(4+6+9)=190$ and 40+60+90 =190.

D 4.

. . . . $\frac{168}{4}$ . . . . . deśa dvātya . . . . pātya jātā 68 recto.

śeshaṁ* 21 . . . . . ekatraṁ 29 . . . . . draṁ 2 . .

. . . . . bari . . . . . .

D 4. [68 *recto.*] Consists of small fragments which probably belong to folio 67. Writing α_2. The phrase *pātya śesham* occurs on some six other occasions (on folios 31, 62, 63, 56).

D 5.

i. . . . . *trai-rāśikena* $\frac{2}{1}$ dine $\frac{3}{1\ 2}$ *draṁ°* $\frac{168}{11}$ *di* . . . . 31 recto.

tīyasya kṛiyate . . . $\frac{3}{1}$ di° $\frac{2}{1\ 2}$ draṁ° $\frac{168}{11}$ dinā phalaṁ draṁ°

$\frac{140}{11}$ prathamena dattaṁ . . . . . saptaḥ dattais samadhanā jātā |

. . . . . sadṛiśaṁ $\frac{77}{11}$ $\frac{294}{11}$ pātya śeshaṁ† $\frac{217}{11}$ dvitīyasya

datta . . . . . 77 eśas sama-dhanā jātā ||

ii. punānyaṁ sarva . . . . . bhā $\frac{4}{1+\ 4}$ dine | draṁ° 15 jīvyā ||

dvitīyasya | bhā $\frac{3}{1\ 3}$ dine | draṁ° . . . . .

D 5. Folio 31 consists of two leaves stuck together and the writing on the two sides differs. The leaf is very ragged.

[31 *recto.*] The writing may be classed as α_2.

i. The example may be restored with some uncertainty: A earns $3\frac{1}{2}$ *drammas* in 2 days, B[2] earns $2\frac{1}{3}$ in 3 days. A gives B 7 *drammas* and this makes their possessions equal. How long had they been earning?

< Since $\frac{3\frac{1}{2}}{2}t-7=\frac{2\frac{1}{3}t}{3}+7$ we have $t=\frac{14}{7/6-5/6}=\frac{14\cdot6}{1\cdot1}=15\frac{1}{11}$ days. >

Proof by the rule of three 2 days : $3\frac{1}{2}$ drammas : : $\frac{1}{11}$ days : $\frac{6}{11}$ drammas

and 3 days : $2\frac{1}{3}$ drammas : : $\frac{1}{11}$ days : $\frac{7}{11}$ drammas |

and $\frac{3\cdot6}{11}-\frac{7}{11}=\frac{18-7}{11}+\frac{7}{11}=\frac{18}{11}$.

ii. Another example of the same kind.

D 5—*contd.*

. . . . *kāraṇaṁ* . . . . chchheda saṁ-guṇe   draṁ° 1 4 ya   31 verso.
1 1
2

draṁ° 1	6	mudgā
	1	
	2	

. . . rdha yutī hṛiti phalaṁ ‖ asya guṇākāro dvayāna

2	1
9 2	13 2

. . . . †uparaṁ guṇaye† adau tāva dva . . . .
. . . guṇaya | 2 | . . . . dvi-nava-bhāgesu .

sūtraṁ ‖

[31 *verso*.] Some of the lower writing shows through and it is very difficult to differentiate. The word *gunākāra* ? 'form of multiplication' occurs again on fol. 42 *verso*.

D 6.

1. . . . . chchheshaṁ ta dviguṇa . tā | 67 verso.
nirgachchha . . . praviśa māne chatvāri dattaḥ
puna dvi-gunaṁ
śūṇya hastaṁ gataṁ tasya kiṁ atra mūladhana syāt |

1	2	bhā°	1	1	2	bhā°	2	2	2	bhā°	3	3	2	bhā°	4
1	1		1	1	1		1	1	1		1	1	1		1

4	2	bhā°	5	1
1	1		1	4

. . . .

D 6. 67. The surface of the leaf is much worn and the writing is in some places rubbed off. The writing is α2.

[67 *recto*.] 1. The example seems to relate to a game at which a certain quantity was staked and eventually all lost. The statement means $1+\frac{1}{2}(2+\frac{1}{2}(3+\frac{1}{2}(4+\frac{1}{2}(5+\frac{1}{2}\cdot\frac{1}{2}))))=<$ [illegible] $>$

D 6—*contd.*

. . . . $\frac{49}{8}$. $\frac{12}{8}$ jāta $\frac{61}{8}$ . . sadṛiśaṁ $\frac{8}{1}$. $\frac{11}{8}$ puna . . 67 recto.

$\frac{16}{8}$ $\frac{61}{8}$ jātā $\frac{77}{8}$ . . . . . sadṛiśaṁ ekasya $\frac{16}{16}$ yutaṁ $\frac{77}{16}$

jātaṁ $\frac{93}{16}$ esha phalaṁ bhavati | . . . .

pratyayaḥ $\frac{93}{16}$ | $\frac{1+}{1}$ $\frac{2}{1}$ $\frac{2+}{1}$ | $\frac{2+}{1}$ $\frac{2}{1}$ $\frac{3+}{1}$ | $\frac{3+}{1}$ $\frac{2}{1}$ $\frac{4+}{1}$ | $\frac{4}{1}$ . . .

ii. huṇḍikā samānayana sūtraṁ ||

dina bhakta viśeshaṁ cha . . . . dvi-guṇaṁ kriyate chaiva

kālam eshāṁ vinirdiśet trai-rāśika vidhānena

. dattaṁ cha pātavyaṁ‡ sūkshme dattaṁ cha tatsamaṁ ||

udāharaṇaṁ || dvi-guṇa . . . .

[67 *verso.*] Worked out by steps [illegible] (5 + [illegible]) = [illegible], [illegible] + 4 = [illegible], [illegible] ([illegible] + 3) = [illegible], [illegible] ([illegible] + 2) = $\frac{77}{16}$, and $\frac{77}{16}+1=\frac{93}{16}$ which is the answer.

Proof ((((([illegible] − 1) 2 − 2) 2 − 3) − 4) [illegible] − (5 + [illegible]) = 0.

ii. This *huṇḍikā sūtra* should be intelligible but it is not yet clear to me.

D 7.

. . . . dviguṇaṁ dviguṇaṁ bhāraṁ . . . . labdhaṁ 28 recto.

14 || puna kṛiya . . . . . .

. . . yet || . . . . . . guṇaye | $\frac{1}{16}$ | $\frac{1}{8}$ | guṇi . jātā . . . . . . 28 recto.

āhuṭva aḍho . . . . guṇa . . . . bhāgasya divardhā χ kim

| $\frac{1}{96}$ | 1 / 1 / 2 | phalaṁ . . . . phalaṁ 5 . . . .

D 7. [28]. Find order 60. Writing ? a4. Not intelligible.

E 1.

. . . . . ekārghaṁ tu paṇyānāṁ eka-dvi-tṛi-chatush-shat **66 recto.**

. . . . . paṇyān imānayaḥ

sthāpanaṁ kṛiyate

1	1	draṁ°	1	2	draṁ°	1	3	draṁ°	1	4	draṁ°	1	6	
1	1		1	1		1	1		1	1		1	1	

. . . .

pratyaya trai-rāśikena **66 verso.**

1	draṁ°	1	rū°	12	draṁ°	phalaṁ	rūpa	12
1		1		1				
1	draṁ°	2	rūpa	6	draṁ°	phalaṁ	rūpa	12
1		1		1				
1	draṁ°	3	rūpa	4	draṁ°	phalaṁ	rūpa	12
1		1		1				
1	*dram°*	4	rūpa	3	dram°	phalaṁ	rūpa	12
1		1		1				
1	*dram°*	6	*rūpa*	2	*dram°*	*phalaṁ*	*rūpa*	12
1		1		1				

E 1. Folio 66 consists of a bad piece of birch-bark containing a large knot. The knot is repeated on folio 53. The find order is 58. Writing is probably α4.

The problem may have been something like this: The rates of purchase are one, two, three, four and six articles for one dramma. What will be the cost of twelve of each ?

The cost of one of each would be $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{27}{12}$ therefore the cost of 12 of each is 27 drammas, and the numbers of articles are 12, 6, 4, 3 and 2.

'Proof by the rule of three'

1	*dram°*	:	1	ru	: :	12	dram°	:	12	ru.
1	„	:	2	„	: :	6	„	:	12	„
1	„	:	3	„	: :	4	„	:	12	„
1	„	:	4	„	: :	3	„	:	12	„
1	„	:	6	„	: :	2	„	:	12	„

E 2.

. . . . 1 yo° 1 di° 6 . . . . **53 recto.**

1	yo°	1	di°	6 . . . .
1				1
2		1		

viśeshaṁ tu tatra gatiṁ

3	2	viśeshaṁ	1
2	1		2

. . . sarva gatI

E 2. Folio 53 resembles fol. 66 in appearance and has the same large knot. Its find order is not known. Writing ? α4. The problems are, however, similar to that on fol. 9 *recto*.

[53 *recto*.] The following conjectural restoration of the problem is offered:

'One goes $1\frac{1}{2}$ yojana in a day and another 6 in 3 days. If the first had a start of 9 yojanas when would the second overtake him ?

Since $\frac{3}{2}t+9=\frac{2}{1}t$ we have $t=\frac{9}{\frac{2}{1}-\frac{3}{2}}=18$ days.'

'Proof by the rule of three': 1 day : $\frac{3}{2}$ yo° : : 18 days : 27 yo° and 27+9=36

1 „ : 2 yo° : : 18 „ : 36 yo°

E 2—*contd.*

yojana | 9 | anena guṇaye | 18 | anena . . . . bhavishyati

pratyaya trai-rāśikena | 1 di° / 1 | | yo° 27 / 1 dina 6 ādau yojana 9 .

. 1 / . 1 | 1 di° / 1 . . yo° | phalaṁ yojana

. . . .	18 yojana	20 dina	phalaṁ yo° 27 / 7	53 verso
	1	20 ghaṭike		
		35* gha° dina		
. . .	27 yo	20	pha° yo 36 / 7	
		20 ghaṭike		
		35* ghaṭike dina		

53 verso.

[53 *verso.*] The following is merely a guess at the problem: One goes 18 yojanas in 96 days and another 27 yojanas in 108 days. The first starts from A and the second from B and the distance AB is 9 yojanas. When will they meet if they go only for $\frac{1}{11}$ or 35 *ghaṭikas* of each day? (60 *ghaṭikas*=24 hours).

In one day they together go $\frac{1}{96}+\frac{1}{108}=\frac{7}{72}$, that is they meet at the rate of 1 yojana in $\frac{72}{7}$ days and actually meet each other in $\frac{9\times 16}{7}$ days $=20\frac{4}{7}$ or 20 days 20 ghaṭikas.

Proof 96 days : 18 yo° : : 20d. 20 gha° : $\frac{80}{7}$ yo° and $\frac{72}{7}+\frac{8}{7}=9$.

108 days : 27 yo° : : 20d. 20 gha° : $\frac{72}{7}$ yo.

E 3.

udā° || śaḍ-vimśas cha tṛi-paṁchāśa ekona-tṛimśe vacha | 58 recto.

dvā-śa . . . śaḍ-viṁśa chatuś-chatvāliṁśa saptati |

chatush-shashṭi nava . . . . . ṁśa naṁtaraṁ |

trir-āśīti ekaviṁśa ashṭa . . . . . pakaṁ |

. . . . 296226447064994 . . . .

E 3. [Folio 58.] Find order not known. Writing ? a4. Possibly two leaves stuck together.

[58 *recto.*] This gives pairs of numbers, first in words and then in figures, thus:

Twenty-six and fifty-three and one less thirty

. . . . twenty-six, forty-four, seventy

sixty-four . . . . . .

eighty-three, twenty-one, . . eight . .

and in figures . . . . . 29 62 26 44 70 64 99 4 . . .

E 3—*contd.*

. . . . . 58 verso.

sthāpanaṁ kṛiyate . . .	1	yuvi	1	sūḍha	1	dṛishya	20
	1		1		1		
. . .	3	maṁ	1	maṁḍa	1	maṁḍe	20
			1		2		
			2				

. . . . . ta datta jātaṁ maṁḍa 2 yu 5 sūḍhe 1 . . . .

[58 *verso.*] There is basis for the following restoration—

A man earns 3 in one day, a young woman 1½ in 1 day and . . . ½ in one day. If 20 earn 20 *maṇḍas* in one day, how many of each will there be?

Let x, y, z be the numbers of each class, then x+y+z =20 individuals

$3x+\frac{3}{2}y+\frac{1}{2}z = 20$ *maṇḍas*

of which the only solution in positive integers is that given in the text, namely x=2, y=5, z=13. This problem known as the 'Hundred Hens' problem in China, and as the *Regula Virginum*, etc., in Europe is noted upon in Part I, §80 (a).

E 4.

. . . . . tṛi-bhāga . . dine tatha | tṛi rūpa paṁchabhi dinai | 21 recto.

eshāṁ da . . . .

rū° 1	rū° 1	rū° 3	dṛishya 100
1	1	1	1
1 di°	1 di°	5 di	
3	2	1	

karaṇaṁ || . . . . . kṛitvā | 3 | 2 | 3 | dri° 100 |
|---|---|---|---|
| 1 | 1 | 5 | 1 |

E 4. Folio 21 consists of 7 scraps of which the largest piece is partly intelligible. The find order is 55 and the writing α1, 4.

[21 *recto.*] Apparently this means: 1 rū° is given or obtained in ½ days, 1 in ¼ day and 3 in 5 days by three separate individuals (or classes) and the total amount given or obtained is 100.

In one day $\frac{1}{\frac{1}{2}}+\frac{1}{\frac{1}{4}}+\frac{3}{\frac{1}{5}}=3+2+\frac{3}{5}=5\frac{3}{5}$ is given, so that one is given in $\frac{5}{5\frac{3}{5}}$ days and 100 in $\frac{500}{28}=17\frac{4}{7}$ days.

E 4—*contd.*

. . . . . vārdhaṁ tritīyasya 21 verso.

jīva-lokāt eshāṁ dīnār . . . . . kasya kiṁ bhavati ‖

2	dī°	3	dī°	4	dī°
1		1		1	
2		2		2	
1	dī°	1	dī°	1	dī°
1		1		1	
2		3		4	

. . . . . parivartanaṁ kṛiyate	10	21	36	dṛi 500
	6	8	10	1

prakshe . . . . . .

[21 *verso.*] Here the main elements of a problem are preserved and the problem is continued on folio 22. The problem probably was to the effect that: A gave $2\frac{1}{2}$ *dināras* in $1\frac{1}{2}$ days, B gave $3\frac{1}{2}$ in $1\frac{1}{4}$ days and C $4\frac{1}{2}$ in $1\frac{1}{3}$ days. In what time would they have given 500 *dināras*?

In one day $\frac{2\frac{1}{2}}{1\frac{1}{2}}+\frac{3\frac{1}{2}}{1\frac{1}{4}}+\frac{4\frac{1}{2}}{1\frac{1}{3}}=\frac{10}{6}+\frac{21}{8}+\frac{36}{10}\ldots=\frac{947}{120}$ is given. Therefore 500 is given in $\frac{500\times120}{947}=\frac{60000}{947}=63\frac{279}{947}$ days.>

Continued on fol. 22 *recto.*

E 5.

. . . . . 473500 / 947 vartita jātā phalaṁ dī 500 ‖ 22 recto.

asya *pratyaya trai-rāśikena*

2	dī°	1	dī°	100000	dī°	phalaṁ dī	60000
1		1		947			947
2		2					
3	dī°	1	dī°	157500	dī°	phalaṁ dī	60000
1		1		947			947
2		3					
4	dī°	1	dī°	216000	dī°	phalaṁ dī	60000
1		1		947			947
2		4					

E 5. [22 *recto*] continues the solution of the example on fol. 21 *verso*.

<The gifts are therefore $\frac{100,000}{947}+\frac{157,500}{947}+\frac{216,000}{947}>=\frac{473,500}{947}$ =500 *dināras.*

'Proof of this by the rule of three' $2\frac{1}{2}$ *dī°* : $1\frac{1}{2}$ days : : $\frac{100,000}{947}$ dī : $\frac{60,000}{947}$ days.

$3\frac{1}{2}$,, : $1\frac{1}{4}$,, : : $\frac{157,500}{947}$,, : $\frac{60,000}{947}$,,

$4\frac{1}{2}$,, : $1\frac{1}{3}$,, : : $\frac{216,000}{947}$,, : $\frac{60,000}{947}$,,

F 1.

i. . . . . dvi-guṇam dvitīyasya prathama . . . . tīya . . | prathamā 22 verso.

chaturguṇaṁ chaiva chaturthe chaiva dattavān cha . . . . . śatam ekaṁ

dvayānvayaṁ || vadasva prathame dattaṁ kiṁ pramāṇāṁ . . sya .

0	2	3	4	dṛishya 200
1	1	1	1	1

†śūṇyam eka-yutaṁ kṛitvā† 1 | 2 | 3 | 4 | . . . †kshepa yuktyā†

phalaṁ || 20 | 40 | 60 | 80 | evaṁ 200 || eshām . . . .

ā° 20	u° 20	pa° 4	rūpoṇā karaṇena phalaṁ 200
1	1	1	

ii. sūtraṁ || yadṛichchha pinyase śūṇye tadā vargaṁ tu kārayet

F 1. [22 *verso.*] i. This appears to be the beginning of a new section. The *sūtra* is lost. Find order 54, writing α4.

The problem was something like this: A certain amount was given to the first, twice that to the second, thrice it to the third, and four times to the fourth. State the amount given to the first and the shares of the others, if the total amount given was 200.

The shares are represented by 0, 2, 3, 4. 'Having added one to the nought' the sum is 1 + 2 + 3 + 4 = 10. Then the proper share of the first is $\frac{200}{10} = 20$. Having added in this value the series becomes 20 + 40 + 60 + 80 = 200.

The proof by the *rūpona* method gives $\frac{((4-1)\frac{200}{10}+20)4}{10} = 200$.

For the method of solution, the *regula falsi*, see Part I, §§71 and 72, and for the *rupona* method see §73. The whole section is dealt with in §87, and the use of the symbol for 'nought' in §60.

ii. The *sūtra* begins "Put what number you please in the empty place (or for the nought)." This is quoted on fol. 23 *recto* and so is *tadā vargam tu kārayet*, etc.

F 2.

i. . . . . cha tṛi-guṇaṁ . . . . . 23 recto.

. . . . . prathamasya tu kiṁ bhavet

0	tadā 2	tadā
1	1	

†yadṛichchhā vinyase śūṇye† . . chchhā | 1 | †tadā vargaṁ tu kārayet†

1	2	2 3	6	prakshipe guṇitaṁ 1	2	6	24
1	1	1 1	1				

. . prakshiptaṁ 33 || dṛishyaṁ vibhajet | 132 / 33 | vartyaṁ jātaṁ | 4 / 1 |

F 2. [23 *recto.*] The find order is 52.

i. The example may be represented by $x+2T_1+3T_2+4T_3=132$. Where T_1, T_2, etc., represent the values of the first, second, etc. terms. Make $x=1$ then the terms are $1+2+6+24=33$ and the proper value of x is $\frac{132}{33}=4$ and the series becomes $4+8+24+96=132$.

All the technical terms here employed are of interest and will be dealt with in due course: *ichchhā* 'an assumed number'; *varga* 'a series'; *prakshepa* 'something thrown in' or 'an interpolation'; *vartya* 'cancelled'; *dṛishya* 'the given number'; etc.

F 2—*contd.*

. . . . . dattaṁ ‖ ato nyāṣaḥ | 4 | 8 | 24 | 96 | . . . .

esha varga krama gaṇitam ‖ atha yuti vargam kṛi . . . .

ii. **sūtraṁ** ‖ kāmikaṁ śūṇye vinyastaṁ tadā chaiva krame guṇam

i. . . . . kṛitvā chaturtha . . . . . 23 verso.

. . . . . prathamasya tu kiṁ bhavet

0	2 1	3 3	12 4	dṛi° 300
1	1 1	1 1	1 1	1

‡kāmikaṁ śūṇye piṇyastaṁ‡ kāmikaṁ 1 ‖ esha ṇyastam . . . .

‡tadā chaiva krameṇa guṇitaṁ‡ | 1 | 2 | 9 | 48 | eshām yu . | 60 |

anena dṛishyaṁ bhājitaṁ

1	300	jātā	5	e . . . . .
60	1			

anena kshepaṁ guṇaye | 5 | 10 | 45 | 240 | . . . . . yuti

varga gaṇitaṁ ‖

ii. **udā°** ‖ prathamasya na . . . . dattaṁ chaivā dhanaṁ |

sa cha dvyārdha yuta dhanaṁ . . . . . .

---

ii. The term *kāmika* is practically synonymous with *ichchhā* or *yadrichchhā* 'what you please'; 'an assumed number.' Bhāskara uses *ishṭa* much in the same way. A good deal of the *sūtra* is quoted on fol. 23 *verso*.

[23 *verso*.] i. The example may be represented by $x+2T_1+3(T_1+T_2)+4(T_1+T_2+T_3)=300$. Put $x=1$ than the series becomes $1+2+9+48=60$ and the proper value of x is $\frac{300}{60}=5$ and we have $T_1=5$, $T_2=10$, $T_3=45$, $T_4=240$ and $\Sigma T=300$.

ii. The example is solved on fol. 24 *recto*.

F 3.

śataṁ chatuś-chatvaliṁśā . . . . . **dattaṁ chaiva chaturguṇam** 24 recto.

kiṁ prathamasya . . . . . .

0 1	2 2	3 3	4 4	dṛi° 144
1 1	1 1	1 1	1 1	1
2	2	2	2	2

F 3. [24 *recto*.] The example may be represented by

$$[x(1+1\tfrac{1}{2})]+[2T_1+2\tfrac{1}{2}x]+[3T_2+3\tfrac{1}{2}x]+[4T_3+4\tfrac{1}{2}x]=144\tfrac{1}{2}$$

Set $x=1$ and the series becomes $\frac{1}{2}+\frac{5}{2}+\frac{9}{2}+\frac{13}{2}=\frac{288}{2}=144\frac{1}{2}$ which is the same as the given sum and therefore $x=1$ is correct.

The phrase marked ** is deleted in the original. The expression "*upare uparaṁ adhe adhaṁ guṇaye*" is obviously quoted from a well known rule relating to fractions: 'numerator should be multiplied by numerator and denominator by denominator.' See also C5, D5.

F 3—*contd.*

. . . . . śūṇyeśu $\begin{matrix}1\\1\end{matrix}$ . . . . . †yutaṁ chaiva guṇaṁ† tataḥ

yutaṁ chaiva guṇaṁ kṛitvā kāraye gaṇa . . . . $\begin{matrix}5\\2\end{matrix}$ guṇaṁ | †upare

uparaṁ adhe adhaṁ guṇaye† $\begin{matrix}10\\2\end{matrix}$ sārdha dv . . yutaṁ . tīya rāśyā guṇaṇaṁ |

sārdhais saptabhi trīṇi $\begin{matrix}45\\2\end{matrix}$ sārdha traya yutaṁ . . chaturtha rāśi

guṇayesh shaḍviṁśatibhi | jātā $\begin{matrix}208\\2\end{matrix}$ sārdha chatvāri yu . . . . .

$\begin{matrix}289\\2\end{matrix}$ evaṁ dṛiśyaṁ | sarvaṁ tadeva jātaṁ

i. . . . . tri-sārdha yu . . . . . 24 verso.

. . chatur-guṇaṁ chaturthena navārdha yutaṁ dattaṁ |

. . *dviśa*tā dvāviṁśādhikā kim atra prathamasya dattāsit

$\begin{matrix}0\ 3 & 2\ 5 & 3\ 7 & 4\ 9\\1\ 2 & 1\ 2 & 1\ 2 & 1\ 2\end{matrix}$ ekatraṁ dattaṁ 222

‡śūnya . . . . datvā‡ | 1 | yuta guṇita yuta krameṇa jātaṁ ||

sthāpā . . . . . $\begin{matrix}5 & 15 & 67 & 357\\2 & 2 & 2 & 2\end{matrix}$ dṛishya 222 prakshepena

jātaṁ 222 || . . dṛiśyāḥ 222 ||

ii. udā° || prathamaṁ na jānāmi | divardha yutaṁ . . . . .

[24 *verso*]. i. The example may be represented by
$[x(1+1\frac{1}{2})]+[2T_1+\frac{1}{2}x]+[3(T_1+T_2)+\frac{1}{2}x]+[4(T_1+T_2+T_3)+\frac{1}{2}x]=222.$
Set $x=1$ and the series becomes $\frac{1}{2}+\frac{5}{2}+\frac{15}{2}+\frac{35}{2}=222.$
The same quotation *śūnya śīkāṁ* . . . *rūpaṁ datvā* occurs on fol. 25 *verso*. See also at the bottom of fol. 26 *recto*.

F 4.

| 0 3 / 1 2 | 2 5+ / 1 2 | 3 7+ / 1 2 | 4 9+ / 1 2 | dṛi° 78 / 1 |

25 recto.

. . . . . yutaṁ jātaṁ | 5 / 2 | dvitīya guṇaṁ | 10 / 2 | . . tṛitīya ekatre guṇitaṁ | yutena | yutaṁ | 10 / 2 | | 23 / 2 | . . yutaṁ | 33 / 2 | guṇitaṁ | 132 / 2 | riṇaṁ jātaṁ | pārya . . eśa *nyāsa* | 5 / 2 | 5 / 2 | 23 / 2 | 123 / 2 | dṛishya 78 / 1 |

. . . . | 156 / 2 | vibhaktavyaṁ | 2 / 156 | 78 / 1 | . . . . .

i. karaṇaṁ | †śūṇya sthāne†.....†rūpaṁ datvā† | 1 | yutā jātā | 5 / 2 |

25 verso.

..... | 15 / 2 | prathamā tṛitīyasya tṛi-guṇaṁ yutaṁ jātaṁ.....

chaturguṇaṁ navārdha yutam jātaṁ | 29 / 2 | ekatra *nyāsa*..... | 5 / 2 | 15 / 2 | 22 / 2 | 29 / 2 | dṛi° 71 / 2 | prakshiptaṁ | 71 / 2 | bhaktaṁ dṛishyaṁ jātaṁ

1.....anena sarvaṁ guṇitaṁ tadeva | 5 / 2 | 15 / 2 | 22 / 2 | 29 / 2 | ekatraṁ ||

eshāṁ aparo vidhiḥ ||

ii. udā° || prathama dhanaṁ dattam najātaṁ kiṁ tu divardha yutaṁ |

tadā dvitīyena dvi-guṇaṁ dattaṁ paṁchārdha hīnaṁ |

tadā tṛitīyena tṛiguṇaṁ dattaṁ saptārdha . .

chaturtheṇa chatur-guṇaṁ navārdha hīnam . . .

dattaṁ ekatraṁ ta . . . . . .

| 2 5 / 1 2+ | 3 gu° 7 / 1 2+ | 4 gu° 9 / 1 2+ | *dṛi 29* / 2 |

F 4. [25 *recto.*] The example may be represented by

$[x(1+\frac{1}{2})]+[2T_1-\frac{1}{2}x]+[3(T_1+T_2)-\frac{1}{2}]+[4(T_1+T_2+T_3)-\frac{1}{2}x]=78.$

Set $x=1$ and the series becomes $\frac{1}{2}+\frac{3}{2}+\frac{9}{2}+\frac{1}{2}^{3}=\frac{1}{2}^{6}$ and $\frac{78}{156/2}=1$.

[25 *verso.*] i. The example, of which only the solution remains, is

$[x(1+\frac{1}{2})]+[2T_1+\frac{5}{2}x]+[3T_1+\frac{1}{2}x]+[4T_1+\frac{1}{2}x]=\frac{9}{2}$, which, when $x=1$, becomes $\frac{1}{2}+\frac{9}{2}+\frac{9}{2}+\frac{9}{2}=\frac{9}{2}$.

ii. The example is $[x(1+\frac{1}{2})]+[2T_1-\frac{1}{2}x]+[3T_1-\frac{1}{2}x]+[4T_1-\frac{1}{2}x]=\frac{9}{2}$.

The solution is given on fol. 26 *recto.*

F 5.

26 recto.

i. karaṇaṁ || †śūnya . . . . . †rūpaṁ datvāḥ† yutam jātaṁ | 5 / 2 |

. . . . . | 5 / 2 | prathama tṛitīyaṁ tṛi-guṇam . . . . . prathamā

chaturthaṁ chatur-guṇaṁ navārdha rahitaṁ | śesham | 11 / 2 | e . . . .

| 5 / 2 | 5 / 2 | 8 / 2 | 11 / 2 | dṛi° 29 / 2 | prakshepa yuktiḥ | 29 / 2 |

. . . . . bhaktaṁ | 2 / 29 | 29 / 2 | jātaṁ | 1 | . . . gunitaṁ tad eva |

evaṁ riṇa rāśī bhavanti |

ii. tṛi-prakāraṁ . . samāptam || śūṇya sthāne rūpaṁ datvā | tadanu

yuktaṁ | guṇita . . . . .

F 5. [29 *recto.*] i. This is the solution of the example given at the bottom of fol. 25 *verso*. Let x=1, then the series becomes $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{1}{2}$ and the correct value of x is $\frac{1}{2}\div\frac{1}{2}=1$.
ii. "The three-fold method is completed," namely, "having put unity in the nought (empty) place; then having added . . .
The *śūnya sthāne rūpam datvā* is quoted on folios 24 *verso*, 25 *recto* and at the beginning of 26 *recto*.

F 6.

26 verso.

atha dvau 4 | 36 asya dalaṁ pha°

athāshṭa 8 | 32 dalaṁ pha° . . . .

16 | 28 dalaṁ pha° . . . .

4 bhu° 36 | 24 | 16 dalaṁ pha° . . . .

24 4 atha *trīni usārā* da . . . . . .

28 4 36 | 20 | 4 asya tri . . . .

32 4 32 | 20 | 8 a . . .

36 4 28 | 20 | 12 puna . . . . .

bhu° 36 24 | 20 | 16 . . . .

F 6. [26 *verso.*] This is, apparently, the beginning of another section, but it is isolated and although there seems to be abundance of material (compared with other leaves) I can make nothing of the problem.

G 1.

i. sūtraṁ 24 — 10 recto.

ii. sūtraṁ || *kṛitvā rūpa kshayaṁ pārtha dhā*nta samguṇa*naṁ tataḥ*
pravṛ*ittir guṇanaṁ tataḥ* . . . . vinirdiset ||

iii. udā° || tṛi-bhāga maladagdhasya tṛi-dhāntasy aiva . . .
ashṭottara-śatāni dattaṁ kiṁ śeshaṁ vada paṇḍita ||

108	1	1	1
1	1	1	1
	3+	3+	3+

kṛitvā rūpa kshayaṁ pārtha† jātā 32 | śesha || prathamab† dhānte
kshayaṁ | 36 | śeshaṁ | 72 | dvitīyab dhānte kshayaṁ | 24 | śeshaṁ 48
tṛitīyab dhānte kshayaṁ | 16 | śeshaṁ | 32 |

pratyayaṁ kṛiyate | sthāpanaṁ

0	1	1	1	bhā°	śeshaṁ	32	phalaṁ mūlā 108 || atha . .
1	1	1	1			1
	3+	3+	3				

sajāti kṛiyā

G 1. Folios 10 to 15 form a fairly well defined section and the leaves are among the best preserved of the manuscript. The 'find order' is 42, 41, 40, 39, ?, 29 and the writing α2. The *sūtra* numbers 24 and 25 occur.

[10 *recto.*] i. The end of the *sūtra* is marked with the usual design and the *sūtra* is numbered 24; so that from 10 *recto* to the end of 15 *recto* consists of one *sūtra* (25) and its illustrative examples.

ii. Of *sutra* 25 the only complete word preserved is *vinirdeset*. It is reconstructed from quotations and fragments of letters. The *sūtra* is the most quoted one in what remains of the original text, the phrase *kṛitvā rūpa kshayaṁ pārtha* occurring some seven times. The last word of this phrase is, however, variously written *pārtha* (fol. 10 *recto*), *pāstham* (10 *verso*), *pāstam* (12 *recto et verso*), *pāstha* (14 *verso*) and is rather curiously omitted on fol. 11 *recto*. This variation is very curious, because the ligatures *rtha*, *stha*, *sta* are so very unlike that the differentiation can hardly be one of carelessness in writing (and the writing is here particularly good). The meaning of the term is still obscure. Dr. Hoernle suggested *prāsta* 'thrown out' or 'wastage'; but I would translate the whole phrase by 'Having calculated for unity the loss per term.' The following is Dr. Hoernle's translation of the *sūtra*—

'Calculate the loss in one; let the instalments of wastage be multiplied together; with the result let the original provision be multiplied; take the result to be the required remainder.'

iii. The example may be rendered:

The third part of the burnt bronze in three instalments (is lost). The amount given was one-hundred and eight. State the remainder, O Pandit.

The solution according to the rule gives 108 $(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})=32$. But proceeding by steps $\frac{108}{3}=36$ and the remainder is 72; $\frac{72}{3}=24$ and the remainder is 48; $\frac{48}{3}=16$ and the remainder is 32.

The proof may be represented by $x^1=\frac{32}{(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})}$

Continued on the reverse.

G 1—*contd.*

i. . . . . | 0 | . . . . . tṛibhi tryashṭa-bhāga saṁyutaṁ . . . 10 verso.
1
1
3+ tadashṭottara-śatāṁ kiṁ | 27 | 1 | 108 | pha° śe° 32
8 | 1 | 1
1
3+ . . . . yadyekasya trayas traya ashṭa bhāga tadā dvā-
1
3+ tṛiṁśānāṁ kiṁ iti 1 | 3 32 phalaṁ 108
1 | 3 | 1
8

ii. udā° || sakṛid dhāntasya lohasya . daśāṁsha kshīyate-s-trayaṁ |
saptate dviguṇā . cha kiṁ seshaṁ vada paṇḍitaḥ || | 3 | 140 |
10

†kṛitvā rūpa kshayaṁ pāstham† iti . . rūpaṁ | 1 | 3 | kshayaṁ kṛitvā
10

jātaṁ śesha 7 | mūlaṁ | 140 | anena guṇitaṁ jātaṁ | 98 | kshayaṁ | 42 |
10 | 1

evaṁ | 140 |

. . . . . | 7 | 1 98 phalaṁ 140 |
10 | 1 1

G 1. [10 *verso.*] i. Gives further proofs of the example on the obverse, namely
$x^1 (1-\frac{1}{3})(1-\frac{1}{10}) (1-\frac{1}{3}) = \frac{7}{10}$ 32, hence x=108 ;
then two proportions in words and figures $\frac{7}{10}$: 1 : : 108 : 32 and 1 : 32 : : 32 : 108.
ii. *Example.*—Of iron one-third refined three-tenths is lost. What is the remainder of twice seventy, tell me Pandit ?
The loss on unity is $\frac{3}{10}$ and the remainder is $\frac{7}{10}$. The original quantity is 140 and $\frac{3}{10}$ of 140=98. The loss is therefore 42 and 98+42=140.
Proof. $\frac{7}{10}$: 1 : : 98 : 140
Continued on fol. 11 *recto.*

G 2.

i. pratyayaḥ 0 . . . . . 11 recto.
1
1
3+
10

ii. udā° || . . palā kṛite pala tṛi-bhāgaṁ kshya vrajati |
ashṭā-daśa . . . . . thatāṁ brūhi ||

G 2. [11 *recto.*] i. Continued from fol. 10 *verso.* 'Proof $x(1-\frac{3}{10})=\frac{7}{10}98$, therefore x=140.'
ii. *Example.*—In purchasing one and a half *palas* the loss is one-third. State what would be the loss on eighteen.
Since $\frac{1}{2}/\frac{3}{2}=\frac{1}{3}$, the loss on unity, the remainder is $\frac{2}{3}$. Now $\frac{2}{3}$ of 18 =14 and the loss is 4.
Proof by the rule of three :— $1\frac{1}{2}$: $\frac{1}{2}$: : 18 : 4 and $\frac{1}{2}$: $1\frac{1}{2}$: : 4 : 18.

G 2—*contd.*

1	3	bhā	18
3	2		1

karanaṁ | addhyardha palaṁ-s-chhedebhya idam | 2 / 9 | †kṛitvā rūpa

kshayaṁ† rūpaṁ | 1 | kshayaṁ kṛitvā jātaṁ | 7 / 9 | 18 / 1 | guṇitaṁ jātaṁ

14	kshayaṁ	4

pratyaya trai-rāśikena ||

addhyardha pala krīte tṛi-bhāgaṁ kshaya gachchhati |

ashṭā-daśa pala krīta kiṁ kshayaṁ vada paṇḍita ||

1 / 1 / 2	1 / 3	18 / 1	phalaṁ	4 / 1

puna tṛi-bhāga divardhaṁ tadā chatubhi × kiṁ iti

1 / 3	1 / 1 / 2	4 / 1	phalaṁ	18 / 1

iii. udā° || chatur-bhāga mala *dagdha* *suvarṇa* śata-paṁ*chakaṁ* |

. . . . . atha pratyay . . . . 11 verso.

0	158	su°	phalaṁ mūla 500 || punar eva prastāra kramaṁ
1	1	to°	
1 / 4+	5* / 1		
1 / 4+	64		
1 / 4+			
1 / 4+			

500 / 1	1 / 1 / 4+	1 / 1 / 4+	1 / 1 / 4+	1 / 1 / 4+	phalaṁ śesha
					tad iti

śesha 158 to° 1 śe° 1 / 64

iii. *Example.*—In refining bronze there is a loss of one-fourth. What would be the loss on 500 *suvarṇas* four times refined ?

The solution is lost. It amounted to < 500 (1—¼) (1—¼) (1—¼) (1—¼) = 158¾ = 158 *suvarṇas*+1½ *tolās*, since 5 tolās = 1 *suvarṇa* >.

Continued on the reverse.

G 2. [11 *verso.*] This appears to have contained five proofs of the example on the obverse, for the present third proof is designated 'the fourth.' The proofs are—

i. Missing.

ii. $x^1(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})=158$ *su*°+1½ *to*° therefore $x^1=500$.

iii. 500 $(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})=x^1$ and $x=158$ *su*°+1½ *to*°.

iv. $x^1=(158$° *su*°+1½ *to*°$)\div(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})$ and $x^1=500$.

v. The first loss is $\frac{500}{4}=125$ and the remainder is 375.
The second loss is $\frac{375}{4}=93\frac{3}{4}=93$ *su*°+3 *to*°+9 *mdsha*. (Since 12 *mdsha*=1 *to*°) and the remainder is 281¼.
The third loss is $28\frac{1}{4}\div 4=7\ \frac{1}{16}$ add the remainder is 210¾.
The fourth loss is $210\frac{3}{4}\div 4=52$ and the remainder is 158½.

G 2—*concld.*

anyaṁ chaturtha pratyayaṁ kṛiyate

0	1	1	1	1	bhā°	śesha	158	phalaṁ	500	‖
1	1	1	1	1			1			
	4+	4+	4+	4			5*			
							1			
							64			

ādyaṁ kshayaṁ 125 śeshaṁ 375 | dvitīye kshayaṁ 93 to° 3 māśa 9

śeshaṁ	281	kshayaṁ	70	śeshaṁ	210	kshayaṁ	52		
	1		5		15		47		
	4		16		16		64		

śeshaṁ	158	eśa sarvatra kartavyā	‖
	13		
	64		

G 3.

. . . . . prastha . . . . . madhunās tathāh 12 recto.

aṁbhasa . . . . . .

†kṛitvā rūpa kshayaṁ pāstam† iti : tatra kshayaṁ : pāstam : iti : tatra kshaya :

rūpaṁ guṇya śeshaṁ	3	3	3	3	4	gadyūti gadyūti gatvā-
	4	4	4	4	1	

t-prasthaṁ pivet . . . . . gadyūti yojanaṁ | chatu prasthai

ādhakaṁ | tadā dhāntaśor gu . . tataḥ | 81 / 256 | āvṛitti pravṛittir-guṇanaṁ tataḥ

| 4 | anena guṇitam jātaṁ | 81 / 64 | eśa maddhva bhāgā bhāge hṛite labdhaṁ

| madnu prastha 1 ku° 1 śe° | 1 / 16 | aṁbha bhāgā prastha 2 kuḍava 2

śe | 15 / 16 | evaṁ | 4 | . . . kuḍavokti prakshepake ādhakā śoḍasha kuḍavā

bhavanti | 16 | ato ma . . . . . śeshaṁ 12

G 3. [12 *recto.*] This is not directly connected with folio 11 but is probably correctly placed here. The find order places it between folios 11 and 13 and it is definitely connected with folio 13. Also it quotes from *sūtra* 25 on folio 10 *recto*. It has the same knot as folio 13.

The example may be conjecturally restored: A traveller goes a journey of 4 *gavyūtis* and takes with him 4 *prasthas* of wine. After each *gavyūti* he drinks 1 *prastha* and then fills up his bottle with water. How much wine and how much water will there be at the end of his journey?

The preliminary part of the solution is rather confused. Possibly the *visarga* marks denote deletion. The general solution is $4.\frac{1}{4}.\frac{1}{4}.\frac{1}{4}=4.\frac{1}{4^4}=\frac{1}{4^3}=1\frac{3}{4^3}$ *prasthas* of wine remain and $2\frac{1}{4^3}$ *prasthas* of water. The number of *gavyūtis* in a *yojana* are mentioned (?2), and the number of *prasthas* in an *ādhaka* are said to be 4 and the number of *kuḍavas* in an *ādhaka* are given as 16. Therefore the wine left over $\frac{3}{4^3}=1$ *prastha* $+1\frac{1}{4^3}$ *kuḍavas* and the water $=2\frac{1}{4^3}=2$ *prasthas* $+2\frac{1}{4^3}$ *kuḍavas* and the sum of these is 4 *prasthas*.

Continued on the reverse.

G 3—*contd.*

i. . . . . prastha kuḍavā | 4 | 3 | śesha chatvāra . . . 12 verso.

kuḍavaḥ 2 2 śeshā cha kuḍavā pītā | ma° | 7 | 9 | puna
1 1 | 1 | 1
4 4 | 4+ | 4

chatvāri kuḍavā bhuktaṁ śeshaṁ | 81/16 | 175/16 | jala bhāgaṁ | madhu kudava

5 1/1 śe° 1/16 | jala kuḍava | 10 1/1 śe° 15/16 | evaṁ kuḍava 16 ||

ii. udā° || datvā śulkaṁ chatur bhāgaṁ ashṭau āṇita kuṅkumā |

chatu śulka śālais tu kiṁ śeshaṁ vada paṇḍita ||

8 1
1 1
4+

karanaṁ | ⟨kritvā rūpa kshayaṁ pāstaṁ⟩ pāstaṁ | 8 3/1 4 | guṇitaṁ

jātaṁ 6 śulke 2 śeshaṁ 6 1 / 1 1 / 4+ anena guṇitaṁ jātaṁ

4 1 2 kshayaṁ 1 1 2 śeshena 4 1 / 1 1 / 2 4+ | datvā guṇita jātā | 27/8

. . . . . .

G 3. [12 *verso.*] i. The solution of the example on the obverse is now done by steps. The original amount of 4 *prasthas* is expressed in *kuḍavas*, namely 16.

Of these 16 *kuḍavas* of wine he drinks ¼ and 12 are left and he adds 4 of water. He then drinks ¼ of wine and there are 9 *kuḍavas* left and the water is made up to 7 *kuḍavas*. Then he consumes ¼=2¼ of wine and there are 9—2¼=7—¼ and the water is made up to 9¼. He then drinks 6/4=1½ and there is left 6¼—1½=5¾ and the water is made up to 7½. There is, therefore, finally ¼+¾=1 *kuḍavas* of wine and 7½=10½ *kuḍavas* of water and these added together give 16 *kuḍavas*. See part I, § 89.

ii. *Example.*—Having given one-quarter as toll at four toll-houses eight of saffron is brought in. State, O Pandit, what is left.

Solution. 8×¼=6 and 2 is paid in toll; 6 (1—¼)=4½ and the loss is 1½: 4½ (1—¼)=3 3/8 and the toll is 1½; 3½ (1—¼)=2⅝ and the last toll is ⅞; and the total toll paid is 2+1½+1½+⅞=5⅛ which leaves 8—5⅛=2⅞.

Continued on fol. 13 *recto*.

G 4.

i. | 8 1 | 1 1 4+ | 1 1 4+ | 1 1 4+ | 1 1 4+ | guṇitaṁ jātaṁ | 81 32 | *punānyaṁ* 13 recto.

| 8 3 4 | 3 4 | 3 4 | 3 4 | phalaṁ | 81 32 | punānyaṁ | 8 1 1 4+ 1 4+ 1 4+ 1 4+ | phalaṁ

| 81 32 | puna pratyayaṁ | 0 1 bhā 1 4+ 1 4+ 1 4+ 1 4+ | śe 81 32 | phalaṁ kuṅkuma 8 ‖

ii. udā° ‖ tṛi-bhāga shaḍ-bhāga pañchāṁśam guḍapiṇḍ āshṭabhārakaṁ |
kiṁ śeshaṁ dattabhir bhavet ‖ . . .

| 8 1 | 2 3 | 5 6 | 4 5 | guṇitaṁ jātaṁ | 32 9 | etat phalaṁ ‖

iii. udā° ‖ chatu ϕ pañchaka lābhena daśa droṇāt prayojita |
tad vai tṛibhis tu kiṁ lābhaṁ katthyatāṁ gaṇakottama ‖

| 10 1 | 5 4 | 5 4 | 5 4 | guṇitaṁ jātaṁ | 1250 64

G 4. [13 recto.] i. Here are four 'proofs' of the example given on folio 12 *verso*.

(*a*) $8\,(1-\frac{1}{2})\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4}) = \frac{81}{32}$.

(*b*) $8.\frac{1}{2}.\frac{3}{4}.\frac{3}{4}.\frac{3}{4} = \frac{81}{32}$.

(*c*) $8\,(1-\frac{1}{2})\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4}) = \frac{81}{32}$

(*d*) $x^1 = \frac{81/32}{(1-\frac{1}{2})\,(1-\frac{1}{4})\,(1-\frac{1}{4})\,(1-\frac{1}{4})}$, whence $x^1 = 8$.

ii. *Example.*—There is a quantity of molasses weighing eight *bhārakas*. What will be left after giving away one-third, one-sixth and one-fifth ?

$8.\frac{1}{3}.\frac{1}{6}.\frac{1}{5} = \frac{8}{90}$ and this is the answer.

iii. *Example.*—By a gain of five-fourths ten *dronas* are obtained. Let it be said, O best of calculators, what will be the gain by three transactions.

(Here the term *lābha* seems to have meaning 'capital + profit,' what is termed the 'mixed quantity' *miśraka* on folio 62.

$10 \cdot \frac{9}{4} \cdot \frac{9}{4} \cdot \frac{9}{4} = \frac{1250}{64} < = 19\frac{34}{64} = 19\ dro^\circ + 2\ āḍ^\circ + 0\ pra^\circ + 2\ ku^\circ >$

For these measures see part I, §109.

Continued on the reverse.

G 4—*contd.*

i. 0 *bhā°* śe° 19 | phalaṁ 10 ‖ 0 | phalaṁ dro° 19 ā° 13 verso.
1 1 1
1 ā° 2 1 | 2 pra° 0 ku° 2
4 4* dro° 4
1 pra° 0 1 | . . .
4 4* ā° pra° 4
1 ku° 2 1
4 ku° 4* prasthi 4

ii. udā° ‖ kasyāpyarjjakasya shashṭhi sva-dalena kshayaṁ gata |
puna vṛiddhyā tṛi-bhāgena sva-pādena tatojjhitaṁ
vṛiddhyā tu paṁcha-bhāgenas tathā vṛiddhi dvayo gataṁ |
kā vṛiddhi . . . . syā kiṁ vā śeshaṁ tad uchyatāṁ ‖

60 1 1 1 1 rūpa lā . . . jātā 36 ‖
1 1 1 1 1
2+ 3 4+ 5

pratyayaṁ punasyaiva 0 1 1 1 1 bhā° 36 | phalaṁ 60 ‖
1 1 1 1 1 1
2+ 3 4+ 5

punānyaṁ pratyayaṁ 60 | phalaṁ 36 ‖ . . . . mūlam na jñāyate
1
1
2+
1
3
1
4+
1
5

0 1 1+ 2 1 3 1 4+ 1 5 phalaṁ .

G 4. [13 *verso.*] i. Continued from the obverse.

(*a*) $x^1=\frac{10\ dro^\circ+2\bar{a}^\circ+0\ pra^\circ+2\ ku}{(1+\frac{1}{2})(1+\frac{1}{3})(1+\frac{1}{4})(1+\frac{1}{5})}=10.$

(*b*) $x^1(1+\frac{1}{2})(1+\frac{1}{3})(1+\frac{1}{4})=19\ dro^\circ+2\ \bar{a}^\circ-0\ pra^\circ+2\ ku^\circ$ <whence $x^1=10$>. See Part I, p. 62.

ii. *Example.*—The capital of a certain banker is sixty. One half of it goes in loss and then he gains by one-third; next he loses one-fourth of it and finally gains one-fifth: so that he has two gains. What is his gain and what is his loss and what the remainder and let that be stated.

Solution: $60(1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5})=36.$

Proofs. (*a*) $x^1=\frac{36}{(1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5})}$, whence $x^1=60.$

(*b*) $60(1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5})=36$ ☞

(*c*) $x^1(1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5})=36$ < whence $x^1=60$ >

G 5.

. . . . . 14 recto.

yasya tanmayatā chakshu . . . . . .

1	1	1
3	4	5

apahṛita śulka piṇḍaṁ 24 ||

karaṇaṁ || †kritvā rūpā kshayaṁ pāsta† | 2 | 3 | 4 | jātu saṁgunya
| 3 | 4 | 5 |

jātaṁ | 2 | 5 | etāvad api rūpa saṁśudhā jātaṁ | 3 | 5 | anena bhaktvā śulka

piṇḍaṁ guṇitaṁ jātaṁ | 40 | eśa piṇḍaṁ

pratyayaṁ | 2 | 40 | 5 | 1 | guṇita jātaṁ 16 śeshaṁ | 24 | evaṁ | 40 |

anyaṁ asya pratyayaṁ | 40 | 1 | 1 | 3 + | 1 | 4 + | 1 | 5 + | phalaṁ 16 kshayaṁ 24 evaṁ 40 ||

ii. udā° || guḍa piṇḍa jñāta tulyoś chatu . . . avye guḍaṁ |

tṛi-chatu ϕ-paṁcha-shaḍ vṛiddhyā chatvāriṁśa (bha*) ve kshaya

---

G 5. [14 *recto.*] i. The find order of folio 14 is unknown. It introduces a variation of the problems given on folios 10 to 13, but it still quotes from the same *sūtra* or a very similar one. The first example can be represented by x (1—½) (1—¼) (1—⅕)=x—24.

Solution: ½.¾.⅘=⅖, 1—⅖=⅗, ⅝×⅗=40 and this is the quantity (*piṇḍaṁ*).

Proof: ⅝ of 40=16 and 40—16=24.

Another proof of this: 40 (1—½) (1—¼) (1—⅕)=16 and 40—16=24.

ii. *Example.*—A known amount of molasses equal to . . four is increased by one-third, one-fourth, one-fifth, one-sixth and then forty is lost . . . . .

No solution is preserved.

i. udā° || ajñātārambha-lohasya tṛi-chatu ϕ-paṁchakā kshaye | 14 verso.

sapta-viṁśati piṇḍasya tṛi-dhānta śeshya dṛishyate |

kiṁ sarvaṁ vada tatvajña kshayaṁ cha mama katthyatāṁ ||

1	1	1	śe°	27
3	4	5		1

G 5. [14 *verso.*] (i) *Example.*—An unknown quantity of lapis lazuli loses one-third, one-fourth, and one-fifth; and the remainder after the three-fold operation on the original quantity is twenty-seven. State what the total was, O wise one, and also tell me the loss.

Solution ½.¼.⅕=1/10; 1—⅘=1/5; 27—⅘=45 and 45—27=18 and this is the loss.

The meaning of *ambha-loha*=lapis-lazuli was suggested by Dr. Hoernle.

G 5—*contd.*

karaṇaṁ | †kritvā rūpa kshayaṁ pāstha† | $\begin{smallmatrix}2\\3\end{smallmatrix}$ | $\begin{smallmatrix}3\\4\end{smallmatrix}$ | $\begin{smallmatrix}4\\5\end{smallmatrix}$ | guṇitaṁ

jātaṁ | $\begin{smallmatrix}2\\5\end{smallmatrix}$ | rūpa kshayaṁ | $\begin{smallmatrix}3\\5\end{smallmatrix}$ | anena śeshaṁ bhaktaṁ śeshaṁ | 27 |

bhaktaṁ jātaṁ 45 asya saptā-viṁśa | pātya seshaṁ 18 | eta kshayaṁ ||

iii. udā° || parikshīṇasya lohasya tṛi-dhāntaṁ paṁcha māśakaṁ |

na jñāyatet pravṛittkāṁ na tu śesha pradṛisyate |

pravṛitti śeshaṁ yo piṇḍaṁ kevalaṁ viṁśatu sthitaṁ |

ajñāta kāṁ pravṛittī syā kiṁ vā śeshaṁ vadaśva me ||

| $\begin{smallmatrix}1\\3\end{smallmatrix}$ | $\begin{smallmatrix}1\\4\end{smallmatrix}$ | $\begin{smallmatrix}1\\5\end{smallmatrix}$ | kritvā

---

ii. *Example.*—Of the loss of iron the third is one-fifth of a māsha. The original quantity is not known and neither is the remainder given; but only the original remainder which quantity stands at twenty. Tell me what is the unknown original quantity and what is the remainder.

This interpretation, however, is by no means certain. The solution is lost.

G 6.

. . . . . pravṛitti bhavet *sakhe* || 15 verso.

| $\begin{smallmatrix}1\\3\end{smallmatrix}$ | $\begin{smallmatrix}1\\3\end{smallmatrix}$ | $\begin{smallmatrix}1\\3\end{smallmatrix}$ | $\begin{smallmatrix}1\\3\end{smallmatrix}$ | śe $\begin{smallmatrix}16\\1\end{smallmatrix}$

karaṇaṁ || dhāntaśo ghātitaṁ tena | †rūpa kshayaṁ kṛitvā† jātaṁ

G 6. [15 *verso.*] There is a suspicion that this is a double leaf. The lenticels on the left side are well-marked but hardly any trace of them appears on the right side. Also the contents are to some extent incongruous.

The example may be represented by $x\,(1-\frac{1}{3})\,(1-\frac{1}{3})\,(1-\frac{1}{3})\,(1-\frac{1}{3})=16$. Now $\frac{2}{3}.\frac{2}{3}.\frac{2}{3}.\frac{2}{3}=\frac{16}{81}$ and $16\div\frac{16}{81}=81$ and this is the original quantity.

Another method by *kala-savarna*. (This term laterally means 'parts resembling one-sixteenth,' but by Mahāvira it is used to denote fractions generally iii. 1). The question is inverted: 'Of iron (refined) four times eighty-one is given. What is the remainder, state, O expert, which is solved by working hard in calculating.'

$$81\,(1-\tfrac{1}{3})\,(1-\tfrac{1}{3})\,(1-\tfrac{1}{3})\,(1-\tfrac{1}{3})=16.$$

"Another proof is made and the original amount is not known."

$$x^1=\frac{16}{(1-\frac{1}{3})\,(1-\frac{1}{3})\,(1-\frac{1}{3})\,(1-\frac{1}{3})}=81 \textit{ pala.}$$

G 6—*contd.*

2	2	2	2		16		81
3	3	3	3	guṇitaṁ	81	bhaktaṁ	16

śeshena guṇaye |

	16		81
śeshaṁ	1	guṇita jātā	1

. . . . pravṛittir ity arthaḥ || athānya

vidhi kalā savarṇe

chatur dhānta . lohasya ekāsīus-cha dattavān

kiṁ śeshaṁ vada dharmajña ya gaṇite kritaṁ śramaṁ ||

81	1	1	1	1		
1	1	1	1	1	phalaṁ śe°	16	
	3+	3+	3+	3+		

puna pratyayaṁ kṛiyate mūlaṁ na jñāyate

0	1	1	1	1	bhā°	śe°	16	
1	1	1	1	1			1	phalaṁ loha pala 81	
	3+	3+	3+	3+				

udā° . . . . . 15 recto,

. . . . . kaśchi yadi śakya tad uchyatām ||

etan me saṁśayaṁ prājñad dhānta ksha*yaṁ* vichāraṇāḥ

2	3	4			
3	4	5	ksha°	śe°	32
					1

karaṇaṁ || dhānta saṁguṇya guṇi*taṁ* jātaṁ | 3 / 5 | rūpaṁ dadyā | 8 / 5 |

bhāge hṛite labdhaṁ bhak..... | 5 / 8 | 32 / 1 | phalaṁ 20 eśa sā pravṛitti |

śesham 12.....32 || paṁcha-viṁśatima sūtraṁ || 25

[15 recto.] Only the end of the formal question is preserved—If thou canst state . . . this is my doubt, O wise man, by examination . . . . . .

The example may have been :— $(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5})=x-r$ and $x+r=32$. From this $\frac{1}{5}x=x-r$, $(1-\frac{1}{5})x=r$, $\frac{1}{5}x+x=32$ and $x=32\div\frac{5}{6}=20$, and $r=\frac{1}{5}.20=12$.

G 7.

i. vibhaktaṁ jātaṁ

2	śe°	10	.	.	.	.	.	.	.	.	.	.	.	.	.	9	16 recto.
9		1														7	

anena guṇitaṁ jātaṁ | 90 | bhāge hṛite labdhaṁ 12 ‖

asya pratyaya trai-rāśikena

7	1	10	pha°	12	
6	1	1		6	
	2			7	

ii. udā° ‖ mākshikag-ghaṭakasyaiva dvi-tṛi-bhāga pravardhitaṁ

dvitīye dvi-paṁchamo-bhāgo tritīye dvi-saptakod*bhavaṁ*

chaturthe dvi-navaṁ-bhāgaṁ evaṁ jāta pala trayaṁ |

babhūvā saulkikai hṛitvā *kiṁ* sarvaṁ vada paṇḍita ‖

2	2	2	2	śe°	3
3	5	7	9		1

dhāntaso . . . . . iti | kṛitvā

G 7. [16 *recto.*] i. The find order is 30 and the writing is α2.4. Only the remnants of a problem: Loss on 1½ is 7/6; what is the original when the remainder is 10? Loss on 1 is ½÷1½=⅓ therefore x ⅓=10 and x=¹⁰⁄₃=12½.

Proof by the rule of three: , : 1½ : : 10 : 12½.

ii. *Example.*—Of a *ghataka* of honey two-thirds is given, to the second two-fifths, to the third two-sevenths, to the fourth two-ninths, till only three *palas* (are left). O Pandit, state how much altogether was taken away by the tax collector.

H 1.

. . . . . sūtraṁ . . . . . 16 verso.

i. idāni suvarṇa kshayaṁ vakshyāmi . . syedaṁ

ii. **sūtraṁ** || kshayaṁ saṁguṇya kanakās tadyutir bhājayet tataḥ

saṁyutair eva kanakair ekaikasya kshayo hi saḥ

iii. udā° || *eka-dvi-tri-chatus* saṁkhyā suvarṇā māshakai riṇai |

eka-dvi-tṛi-chatus saṁkhyā rahitā sama-bhāgatāṁ ||

sthāpanaṁ kṛiyate | eshāṁ

1+	2+	3+	4+
1	2	3	4

karaṇaṁ || †kshayaṁ saṁguṇya kanakādibhi† kshayena saṁguṇya jātaṁ

| 1 | 4 | 9 | 16 | ... | esha yuti | 30 | kanakā yuti 10 anena

bhaktvā labdhaṁ . . . .

H 1. [16 *verso.*] i. The end of a *sūtra* is marked but the number is not preserved (probably 26) and then a new section is introduced by the remark—"Now I shall speak about *suvarna kshaya.*" It should be noted that Mahāvira uses the term *kshaya* as synonymous with *varna* in his section (vi, 169ff) on *suvarna kuṭṭikāra.* In our text there seems to be some confusion about the meaning of *kshaya* which here really means *varna* or 'quality' although the author obviously thought it denoted a loss. Mahāvira's rule is—

Kanaka kshaya saṁvargo miśrasavarṇāhritaḥ kshaya jñeyaḥ |
paravarna pravibhaktaṁ suvarna gunitaṁ phalaṁ hemnaḥ || 169 ||

"It should be known that the products of gold *kshaya*, when divided by the mixed gold gives rise to the *kshaya*. When divided by the last *varna* (=*kshaya*) and multiplied by the gold gives the corresponding quantity of gold."

ii. *Rule.*—Having multiplied the parts of gold with the *kshaya* let this sum be divided by the sum of the parts of gold. The result is the average *kshaya*. This means $f = \frac{f_1 g_1 + f_2 g_2 \ldots f_n g_n}{g_1 + g_2 \ldots g_n}$ where f denotes *kshaya* and g gold.

iii. *Example.*—$f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$ and $g_1 = 1$, $g_2 = 2$, $g_3 = 3$, $g_4 = 4$ therefore $f = \frac{1.1+2.2+3.3+4.4}{1+2+3+4} = \frac{30}{10} = 3$.

Continued on fol. 17 *recto.*

H 2.

i.

1	1						
10	30	4	pha°	ma°	śe°	12	17 recto.
1	1	1				1	

ii. udā° || eka-dvi-tṛi-chatus *saṁkhyā* suvarna projjhitā ime

māśakā dvi tṛitām chaiva chatu *saṁkhyā* paṁchakarāṁśakaṁ

kiṁ kshayaṁ

1	2	3	4
1	1	1	1
2	3	4	5

H 2. [17 *recto.*] i. The remnant of a proof of the example given on 16 *verso.*

. . . . . .

10 : 30 : : 4 : 12, i.e., $\Sigma g : \Sigma fg : : g_4 : g_4 F$.

ii. *Example.*—Gold one, two, three, four; 'abandoned' the following *māshakas* one-half, one-third, one-fourth and one-fifth.

$F = \frac{\frac{1}{2}.1+\frac{1}{3}.2+\frac{1}{4}.3+\frac{1}{5}.4}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = \frac{109}{60} = 10 = \frac{10}{60}$.

'Proof by the rule of three' $\Sigma g : \Sigma fg : : g_4 : g_4 F$.

H 2—*contd.*

karaṇaṁ || †kshayaṁ saṁguṇya kanakà† eśa sthāpayate |

1	2	3	4
2	3	4	5

†tad yutir bhājayet tataḥ† hara sāsye kṛite yutaṁ | 163 / 60 | † saṁyutai χ

kanakair† bhaktvā tadā kanaka | 10 | anena bhaktaṁ jātaṁ | 163 / 600 | eśa

ekaika suvarṇasya kshayaṁ ||

pratyaya trai-rāśikena . . .

10 / 1	163 / 60	1 / 1	*pha*°	163 / 600
10 / 1	163 / 60	2 / 1	*pha*°	163 / 300
10 / 1	163 / 60	3 / 1	*pha*°	163 / 200
10 / 1	163 / 60	4 / 1	*pha*°	163 / 150

17 verso.

krameṇa dvaya māshādi uttare eka hīnatāṁ |

suvarṇaṁ me tu sammiśrya katthyatāṁ gaṇakottama ||

sthāpaṇaṁ	4+ / 5	5+ / 6	6+ / 7	7+ / 8	8+ / 9	9+ / 10	1+ / 2	2+ / 3	3+ / 4

†kshayaṁ samguṇya† jātaṁ | 20 | 30 | 42 | 56 | 72 | 90 | 2 |

6 | 12* | eśāṁ yuti | 330 || *kanakānāṁ* yuti 45 | anena bhaktvā

labdhaṁ | 330 / 45 | paṁcha-daśa bhāge chchheda kṛiyate | phalaṁ | 7 / 1 | 6e° | 1 / 3 |

eśaa ekaika māśaka kshayaṁ |

pratyaya trai-rāśikena | 45 / 1 | 330 / 1 | 1 / 1 | phalaṁ | 22 / 3 |

evaṁ sarveshāṁ *pratyayaṁ*

H 2. [17 verso.] I do not understand the problem but it is explained by Dr. Hoernle in the *Indian Antiquary* of 1888 (Vol. XVII, p. 43).

The solution is $F = \frac{5.5+6.5+7.6+8.7+9.8+10.9+2.1+3.2+4.3}{5+6+7+8+9+10} = \frac{330}{45} = 7\frac{1}{3}$.

Proof by the rule of three— 45 : 330 : : 1 : ψ and 'so for all of them.'

* Inadvertently omitted in the manuscript.

H 3.

18 recto.

i. (sūtraṁ) || *aprāpta saṁguṇā kati* *kaṁchanāni tatojjhitaṁ*

. . . . . .

kāṁchanai yad bhave labdha sa kshaya jñāta māśaka ||

ii. udā° || eka-dvi māshako prāpto dvau cha prāptaṁ cha paṁchabhi |

trayaś cha katibhiϕprāpta shaḍ eva . ni kevalaṁ |

chaturbhi māshakair hīṇaṁ kaṭi dṛishṭvā mayā sakhe |

trayaś cha katibhiϕprāptā suvarṇāṁ maśako vadaḥ |

1	2	3	6
2	5	0	4+

karaṇaṁ || †aprāpta saṁguṇā kaṭid† iti | 6 / 1 | *aprāpta kati* chatvāra | 4 |

saṁguṇya jātaṁ | 24 | †kāṁchanāni tatojjhitaṁ† dvābhyāṁ *e*ka paṁchabhi

dvayaṁ saṁguṇya jātaṁ 2 | 10 / 1 | tad yuti 12 | hitvā 2 . . . .

hitvā jātaṁ śeshaṁ 12 || aprāpta gaṇḍikai . . . .

H 3. [18 *recto.*] i. The *sūtra* is largely restored from the quotations given in the solution below. The application of the terms *aprāpta* and *kaṭi* are not at all clear; but given that

$F = \frac{f_1 g_1 + f_2 g_2 + f_3 x}{g_1 + g_2 + x}$ then the *sūtra* states that $x = \frac{F.\Sigma g - (f_1 g_1 + f_2 g_2)}{f_3}$.

ii. *Example.*—Māshakas of one and two, gold of two and five, māshakas of three and gold unknown. All that is known is the sum of *māshakas*, six; and the average *māshaka* four. State the *māshaka* of the unknown gold.

Statement $f_1=1$, $f_2=2$, $f_3=3$; $g_1=2$, $g_2=5$, $g_3=x$; $F=4$.

Solution $x = \frac{4.6-(2.1+5.2)}{3} = \frac{24-2-10}{3} = \frac{24-12}{3} = 4$. .....

18 verso.

. . . . . ashṭa-viṁśatima sūtraṁ

i. sūtraṁ || ūnais saṁguṇya kanakā tat piṇḍaṁ cha viśodhayet

suvarṇa kanakābhyastā rāśi sheshaṁ vibhājayet

aprāpta gaṇḍika śeśa śuddhena kanakena tu |

yal labdhaṁ tat pramāṇaṁ tu gaṇḍikā yā vinirdiset ||

H 3. [18 *verso.*] The end of the 28th *sūtra* is marked.

i. *Rule.*—Having multiplied together the (known) gold pieces and their *varṇas* determine the sum of that. Divide the remainder of that quantity and the sum of the product of the average *varṇa* and known gold by the difference between the average *varṇa* and the *varṇa* of the unknown gold. That which results consider to be the measure of the unknown gold.

This may mean, for example, that if $F = \frac{f_1 g_1 + f_2 g_2 + f_3 x}{g_1 + g_2 + x}$ then $x = \frac{(f_1 g_1 + f_2 g_2) - F(g_1 + g_2)}{F - f_3}$.

H 3—*contd.*

[ii] udā° || eka-dvi-tṛi-chatus saṁkhyā aprāpta māśakāni tu

eka-dvi-tṛi-chatus saṁkhyā ekatrāvartitā kilaḥ

gaṇḍikā jñāta kanakā ūnaikā daśa māshakai |

aprāpta jñāta kanakai pra . . . . . yah

1	2	3	4	0
1	2	.	.	.

karaṇaṁ . . . . . .

ii. The example is not understood.

J 1.

30 recto.

. . . . . .

sūtraṁ | eka yuta nara . . . . sarvash shaḍbhi pa . . . .

. . . . . anena labdhaṁ . . . . hītā prathaṁ

36	42	48	54	6
. .	. .	78	7 .	

30 verso.

. . . . . sadṛiśa kṛi . . . . bhāga hāraṁ kṛiyate | 234 / 70 | . . .

. . . tulāḍhe | 3 / 24 / 70 | mudgāḍhe | 1 / 47 / . . | . . . . . kṛiyate

J 2. [Folio 30.] Find order 32. Writing *a*4. By appearance this fragment and fol. 28 perhaps belong to the same leaf. See also fol. 31.

[30 *recto.*] A restoration is suggested in *part* I, §78, vii, but I doubt its being correct.

[30 *verso.*] We have $\frac{3}{8}\frac{1}{8}=3\frac{1}{8}$ and $3\frac{1}{8}\div 2=1\frac{1}{16}$. The term *mudga* 'a kidney bean' occurs also on folio 31. See also *Līlāvatī*, §97.

J 2.

65 recto.

etat-kāla . . . . timanushyā ya . . . . . lagyanti . . .

apara prashṇaḥ

yady eka purushasya drammāsh-shaṭ . *triṁśa*bhir dinai jīva-lokā | tat kāryaṁ prastutaṁ . ssaptatīnāṁ . . . . . pāka rākshakānāṁ drammaish-shaḍbhi kati dinā jīva-lokaṁ bhav*ati* .

karaṇaṁ | ādau tāva yady ekapurushasya drammāsh-shaṭ triṇśabhi - - - jīvyāh | tat saptatīnāṁ kiṁ

1	pu°	dram°	6	30	di°	70	pu°	phalaṁ . . . .
1			1			1		

drammā . . trīṇi śata-sā . . . . . .

J 2. [65 *recto.*] Folio 65 consists of two leaves stuck together. The *verso* side has been definitely placed as C 4. The writing is here *a*4. The find order is unknown.

[Example:—If a man requires six drammas for his livelihood for 30 days, for how many days will 70 men (guards of a fort?) live on six drammas? The details are, however, uncertain.—K. N. D.]

J 3.

. . . . drammā ashṭa dvā-chatvāliṁśabhir dinai | tat saptati . . . . 41 recto.

ya 42 dine | draṁ° 8 jīvyā | 70 purushā 42 . . .
1 1 1

drammā 560 || yadi paṁcha-śata-śashṭyādhika . . . . dva-chatvāliṁśabhi

tad drammai ashṭabhi kati dinā . .

. . . . 2 adhe dāpaye dattaḥ | 17 adhenopari saṁ . . . uparima 41 verso.
8*
2
3

rāśī dvaya guṇaye | 51 . . . . . ; . upari yukta kṛiyate eka-
6*
2
3

paṁchāśānāṁ 51
6

sthāpaṇaṁ | 1 53 . | phalaṁ ā 17 ṭri . 2

J 3. Folio 41 is much damaged and the illustration (Plate XXVIII) suggests a double leaf; but the illustration is deceptive, for the cause of the uneven colour is the presence of gum on the original leaf. The find order is unknown: writing ct.

[41 *recto.*] This is undoubtedly closely connected with fol. 65 *recto* and the repair of fol. 41 and the separation of the two parts of fol. 65 would possibly make both intelligible.

[41 *verso.*] Not understood. *Possibly the 8 and 6 are change-ratios.

K

i. *udā°* || ko rāsi paṁcha yutā *mūladaḥ* sā rāśis sapta hīna 59 recto.

mūlada ko so rāśir iti prashṇaḥ

0	5	yu°	mū°	0	sā	0	7+	mū°	0
1	1			1		1	1		1

karaṇam | †yuta hīnaṁ cha-m-ekatvaṁ† | 12 | tad dalaṁ | 6 | dvi

hṛiṇaṁ | 4 | dalaṁ | 2 | vargaṁ | 4 | †hīne yutiṁ cha kartavyā† |

hīnaṁ | 7+ | anena yuti | 11 | eśa sā rāśi || asya pratyānayaṁ *kṛiyate*

11	yu°	5	mū°	4	11	7+	mū°	2
1		1		1	1	1		1

paṁchāśama sūtraṁ 50

ii. sūtraṁ | gavāṁ viśesha kartavyaṁ dhanaṁ chaiva puna . .

. . . . . .

K [59]. The find order is unknown but the *sūtra* number is 50 and it probably originally preceded fol. 60. The reverse is blank, which possibly means that there are portions of two leaves stuck together.

(i) *Example.*—What number with five added is a square and that same number with seven subtracted also being a square? What is that number? is the question.

Statement $x+5=\square^1$, $x-7=\square^1$.

Solution <$x=[\frac{1}{2}(\frac{5+7}{2}-2)]^2+7=11$ by steps thus>: having combined the added and subtracted numbers $5+7=12$; that halved $=6$; two subtracted 4; halved 2; squared 4; then the subtractive number (7) is to be added and by the addition of this $4+7=11$ and this is the required quantity.

Proof: $11+5=4^2$, $11-7=2^2$. See Part I, §81.

(ii) There appears to be a reference to this fragment on fol. 60 *recto* where *sūtra* 51 is closed.

L 1.

. . . . . 60 recto.

i. . . ekona-vimśatima | gāvo 10 / 1 | rūpa 8 / 1 | . . . | . . . vivaritāsti ||

eka paṁchāsama sūtraṁ 51 ||

ii. sūtraṁ || āya vyaya viśeshaṁ tu vibha*jya* dṛishya saṁguṇaṁ |

yal labdhaṁ sā bhavet kālam ayam prashne . ya vidhi ||

iii. udā° || dvi-dine ārjaye paṁcha tṛi-dine nava bhakshaye

bhāṇḍāgāraṁ tasya tṛiṁśā kiṁ kālaṁ ārja bhakshaṇam ||

| dī° 5 / dī° 2 | dināra 9 / dina 3 | *dri*° 30 |

karaṇaṁ | †āyā vyaya viśeshaṁ tu† | tatrāyaṁ | 5 / 2 . . . . . .

. . . . . .

L 1. [60.] Writing a2. Notice the 'sickle's. Find order unknown. Connected with fol. 59 on one side and folios 61—63 on the other. Folios 60—63 form a fairly definite section (L) relating to earning and spending.

[*60 recto.*] (i) This fragment is connected with the *sūtra* at the bottom of fol 59, but very vaguely.

(ii) *Rule.*—The known quantity is divided by the difference between the expenditure and earning. This result is the time .... This means $t = \frac{a}{1-e}$

(iii) *Example.*—In two days one earns five, in three days he consumes nine. His store is thirty. In what time will his earnings be consumed?

Solution: $t = \frac{30}{1-6/2} = 60$ and the amount earned in this time is $\frac{5}{2}$ of 60 = 150 dīnaras.

i. . . . . bodi | phalaṁ 180 | dvāpaṁchāśama sūtraṁ 52 || 60 verso.

ii. sūtraṁ | aha dravya harāśauta tad viśeshaṁ vi*bhājayet*

yal-labdhaṁ dviguṇaṁ kālam° dattā sama-dhanā prati ||

[*60 verso.*] (i) Remnant of proof of the example on the obverse. The complete proof probably was:—

2 days : 5 *dīnāra* : : 60 days : 150 *dīnāra*

3 days : 9 *dīnāra* : : 60 days : 180 *dīnāra*

and 180—150=30.

(ii) *Rule.*—(If one earns e_1 in d_1 days and another e_2 in d_2 days and the first gives g to the second then $\frac{e_1}{d_1} t - g = \frac{e_2}{d_2} t + g$ and) $t = \frac{2g}{\frac{e_1}{d_1} - \frac{e_2}{d_2}}$

L 1—*contd.*

iii. udā° | tṛi-dine ārjaye paṁcha bhṛitako-m-eka paṇḍitaḥ

dvitīyaṁ paṁcha divase rasam ārjayate budhaḥ

prathamena dvitīyasya sapta dattā *nidhānataḥ*

datvā sama-dhanā jātā kena kālena katthyatāṁ

5 rū	6 . . . .
3 di	5 . . . . .

See *Indian Antiquary*, XLII (1888), pp. 41, 44; but in 1915 Dr Hoernle sent me the following note:—"The textual difficulty was not fully understood by me: the text is badly corrupted, a portion (the 2nd *pāda*) has dropped out, and another (the 1st *pāda*) has been mixed up with the commentary. The real text of the first *pāda* is quoted in obverse line 8 of the next folio, in the commentary of the second example of the *sūtra*, and the missing part of the second *pāda* must be supplied from obverse lb. 4 and 5 of *sūtra* 52; which is merely a variant of *sutra* 53. The latter *sūtra* should really run as follows:—

ahadravya vicesham cha vibhajya datta samgunam |
yal-labdham dvigunam kālam datta sama-dhanā prati ||

i.e., "the difference of the daily earnings, having divided (invested), is multiplied with the given amount: the result being doubled is the time; the given amount goes towards making the possessions equal."

(iii) *Example.*—In three days one pandit earns a wage of five and a second wise man earns six (*rasa*) in five days. The second is given by the first seven from his store and by this giving their possessions become equal. Let it be stated in what time.

Solution: $t = \frac{3 \times 7}{6/5} = 30$.

L 2.

i. . . . . anena kālena sama-dhanā bhavanti ‖ 61 recto.

pratyayaṁ trai-rāśikena kṛi*yate*

3	5	30	pha°	50	prathame dvitīyasya (s) sapta dattā |	7
1	1	1				
5	6				śeshaṁ 43 | . . . . . 43 |	
1	1	30		36		
					43 | ete sama-dhanā jātā ‖	

L 2. [61 *recto.*] i. The end of the solution of the example given on the *verso*.
Proof by the rule of three: 3 : 5 : : 30 : 50 and 5 : 6 : : 30 : 36 and 50 − 7 = 43 = 36 + 7.

L 2—*contd.*

ii. udā° ‖ rājaputro dvayo kechi nṛipatis sevya santi vaiḥ
mekāsyāhne dvayash shaḍ *bhāgā* dvitīyasya divardhakam |
prathamena dvitīyasya daśa dīnāra dattavān
kena kālena samatāṁ gaṇayitvā vadāśū me ‖

13	3	dattaṁ	10
6	2		1

karaṇaṁ ‖ †aha-dravya viśeshaṁ cha† | tatva

---

ii. *Example.*—Two Rājputs are the servants of a king. The wages of one are two and one-sixth a day, of the second one and one-half. The first gives to the second ten *dīnāras*. Calculate and tell me quickly in what time there will be equality. (*Indian Antiquary*, 1888, p. 44).
Statement: $\frac{1}{2}$, $\frac{1}{6}$, given 10.
Solution: The difference of the daily earnings. . . . . .
Continued on the reverse.

i. . . . . | 1 | 13 | 30 | pha 65 | prathamena dvitīyasya . 61 verso.
1 | 6 | 1 |
1 | 3 | 30 | pha 45 | . . . . r dattā jātā |
1 | 2 | 1

55 | 56 ‖ sama dhanā jātā ‖

ii. sūtraṁ tri-paṁchāsamaḥ sūtraṁ 53 ‖

sūtraṁ ‖ *vikra*yena kraya*ṁ* bhājya*ṁ* rūpa hīna*ṁ* punar bhajet
lābhena guṇaye tatra nivī bhavati tatra cha ‖

iii. udā° ‖ dvibhi x kṛiṇāti *yas sa*pta vikṛiṇāti tribhish shaṭ
ashṭā-daśa bhaved lābhā kā nivī *tatra* kattbyatāṁ ‖

7	6	18	lābhā
2	3	1	

karanaṁ | †vi . . . . . .

L 2. [61 *verso.*] i. Proof of example on the obverse—
1 : $\frac{1}{2}$: : 30 : 65
1 : $\frac{1}{6}$: : 30 : 45 and 65—10 = 45 + 10.
ii. The rule means $C = \frac{p}{s/a-1}$ where C is the capital, *p* the profit, *c* the rate of purchase and *s* the rate of sale.
iii. *Example.*—One buys 7 for 2 and sells 6 for 3 and 18 is his profit. What was his capital?
Solution.—$C = \frac{18}{\frac{7}{2}-1} = 24$. The proof is given on folio 62 *recto.*

L 3.

nivī jātā | sya pratyaya *trairāśik*ena ‖ 62 recto.

yadi dvibhis sapta labhyate | tadā chaturviṁśatibhi χ kiṁ |

2	7	24	phalaṁ rū° 84 ‖
1	1	1	

i. asya vikrayaṁ kṛiyate | yadi shaḍbhi traya . . labhyate tadā chaturāśītibhi χ kiṁ |

6	3	84	phalaṁ 42	mūlaṁ 24	pātya śeshaṁ 18 eśa lābhāḥ
1	1	1	

chau-paṁchāsama sūtraṁ 54.

ii. sūtraṁ ‖ vikrayaṁ bhājaye chaiva guṇayet kraya piṇḍatāṁ |

rūp*one* mūla guṇaye labdha lābhaṁ cha prāpyate ‖

iii. udā° ‖ dvibhi krīṇāti yas sapta vikrīṇāti tribhish shat . . . . . .

mūlā cha . . . .

L 3. [62 *recto.*] i. Continued from folio 61 *verso.*

"If for two 7 are obtained, then what for twenty-four ?"

2 : 7 :: 24 : 84 articles.

Again "If by six three are obtained then what for eighty-four ?"

6 : 3 :: 84 : 42

and the original quantity was 24 and the difference 42 − 24 = 18.

ii. The rule means p = C (c/s − 1)

iii. *Example.*— Articles are bought at 7 for 2 and sold at 6 for 3.

i. . . . . | 2 | 7 | 24 | pha° 84 | atha vikrayaṁ | 6 | 3 | 84 | 62 verso.
| | 1 | 1 | 1 | 1 | | 1 | 1 | 1 |

pha° 42	. . .	24	pātya śeshaṁ 18	esha lābhaṁ ‖
1				

paṁcha-paṁchāsama sūtraṁ 55

ii. sūtraṁ ‖ vikrayaṁ bhajaye chaiva guṇayet kraya piṇḍavat

vibhaktaṁ sa cha kartavyaṁ guṇaye miśrakaṁ budhaḥ

yal labdhaṁ sā bhaven mūlaṁ *yat*ch . chheshaṁ lābha piṇḍatāṁ ‖

L 3 [62 *verso.*] i. *Solution.*—Continued from the obverse ; $p = 24\left(\frac{1}{2} \div \frac{6}{7} - 1\right) = 18$.
Proof.—2 : 7 :: 24 : 84 and 6 : 3 :: 84 : 42 and 42 − 24 = 18 is the profit.

ii. *Rule.*—$C = \frac{M}{C+s}$ where M = C + p is called the 'mixed' quantity.

L 3—*contd.*

iii. udā° ‖ tṛibhiś cha labhater ashṭau chaturbhiś cha vikrayaṁsh shaṭ
sa mūla lābham utpaṇṇa śataṁ śashṭi vimiśritaṁ |
kiṁ mūlaṁ kaścha lābhaṁ cha kathayed gaṇakottamaḥ ‖

8	6	miśra 160
3	4	1

karaṇaṁ | †vikrayaṁ bhājaye chaiva guṇayet†

. . . . . .

iii. *Example.*—Eight articles are obtained for three and six are sold for four. The sum of the capital and profit is one-hundred and sixty. State, O best of calculators, what was the capital and what is the profit
The solution is lost except for the first quotation, but part of a proof is given on folio 63 *recto*. The solution was $<C=\frac{160}{\frac{1}{3}+\frac{1}{4}}=90$ and the number of articles bought was $\frac{1}{3}$ of 90=240 >.

L 4.

i. . . . . | 8 | 3 | 240 | *phalaṁ 90*
1 | 1 | 1 | 1

. . . . . 63 *recto.*

6	4	240	phalaṁ 160	mūlaṁ	90	pātya śeshaṁ	70
1	1	1	1				

ii. *shat* pañchāśama sūṭraṁ 56

‖ vikrayaṁ cha vibhaktavyaṁ guṇitaṁ kraya rāsivat
kṛitvā rūpa kshayaṁ chaiva vibhaktaṁ mūlam *āpnuyāt*

iii. udā° ‖ pañchabhiś chatu vargaṁ tu gṛihitaṁ kena mānava
. . kenash shat vikrītaṁsh shaṭ pañchaśa riṇaṁ kṛitaṁ |
krava vikraya saṁguṇya nivis tasyaiva kathyatāṁ ‖

16	6	riṇaṁ 56+
5	1	1

. . . . . bhājaye chaiva 1 . . . . . .
6

L 4. [63 *recto.*] i. Proof of example given on folio 62 *verso.*
8 3 : 240 : 90 and 6 : 4 : : 240 : 160 and 160=90+70.

ii. The rule means $C=\frac{1}{1-a/s}$ where l is the loss sustained, *i.e.*, having investigated the selling rate multiply with the purchase rate and having subtracted from unity divide— and the capital is obtained.

iii. *Example.*—With five four squared are obtained by some man. For one six are sold and fifty-six is the loss. Calculating purchase and sale let his capital be stated.

The solution is $<C=\frac{16}{1-\frac{56}{120}}=120$ and the number of articles is $\frac{1}{6}$ of 120=384>.

L 4—*contd.*

i. punāsya vikraya $\frac{6}{1}$ $\frac{1}{1}$ 384 phalaṁ 64 | mūlaṁ 120 | 63 verso.

chatush shashṭi pātya śeshaṁ | 56 | eśa riṇaṁ kṛi.

saptā-paṁchāśama sūtraṁ 57.

ii. sūtraṁ || vastra śulkaṁ yad bhavati tada . hṛita vastrataṁ |

trai-rāśika vidhānena śulka vikraya tatvataḥ ||

iii. udā° || paṭasya śulka viṁśāṁśaṁ ka . . . . tṛis-śataṁ |

paṭa-kānāṁ paṇa kṛite dvau patau hṛita śaulkikau |

. . *mūlyaṁ* paṇa daśas teshāḥ kiṁ mulyaṁ . . . . .

L 4. [63 *verso.*] Proof of example on the obverse : $<\frac{1}{6}$: 1 : : 384 : 120>, then with the selling rate 6 : 1 : 384 : 64 and 120—64=56.

ii. *Rule.*—That which is the tax on cloth is taken in cloth : by the method of the rule of three tax and sale alike.

iii. The example is not understood but reads something like this : The tax on a piece of cloth is one-twentieth part. Some one sells three-hundred. On the pieces being brought to market, two pieces are taken by way of tax : ten is (?) the selling price. What is the value ?

M 1.

i. 1 20 rakti | dhā° 1 su° 1 chhe° 80* rakti-su° rakti 1 pha° dha° 4 20 recto.
1 1 1 1 1 1
4
a° 0
ya° 4 ya° 1 *pā°* 3 *mū* 1
1
4

puna tṛitīyasyaiva 2 20 1 1
1 1 3 1
4 1 1 chhe° ½*
2 2
1 chhe° 1*
4 4

. . . . chhedaṁ 6 dhā°-dra° pha° dhā° 4 ya° 1 . . . pă° 2 mū° 1 ||

suvarṇasya māṇaṁ samā . .

ii. udā° || sa pañcha nava bhāgāṇi dināni trayo-daśaḥ

. . . . . ṇāṁ kiṁ ||

M 1. [20 *recto.*] Section 'M' begins. Writing β.

i. A fragment of a solution or 'proof'. There were at least three statements, of which the second is 1½ of 20 *rakti* : 1 *dhū* +4½ *yū* : 1 *su*° +1 *ra*° : 4 *dhū*° +1 *yū*° +3 *pū*° +1 *mū*° <or 25 *ra*° : 2025 *mū*° : : 81 *ra*° : 6561 *mū* >.

Then a similar statement of the third (restored) 2½ of 20 *rakti* : ½ *dra*° +½.½ *dhū*° +½ *ya*° : 1 *su* +1 *ra*° : 4 *dhū*° +1 *ya*° +1 *ka*° +2 *pū*° +1 *mū*° <or 45 *ra*° : 3625 *mū*° : : 81 *ra*° : 6525 *mū*°>.

The numbers marked with asterisks are change-ratios. See Part 1, §§ 103-104; and § 110 for the measures employed.

ii. *Example.*—Too mutilated to restore.

i. mū 12000 . . . . . 20 verso.

udāharaṇaṁ | sarposhṭā-daśa hasto praviśaty ārdhāṅgulaṁ

sa nava bhāga . . ti ekaviṁśati bhāgaṁ mapaharaṁti |

pratidinenaḥ kiṁ kālena vilaṁ saṁprāpyate ||

1 1 1 1 18 chhe° 24* aṁ° ha° *phalam ra*° 2 *mā*° 4 *di* 10½
2 21+ 1 360 1 1
1
9

udāharanaṁ | kiṇa x kilārdhaṅgulaṁ divase divase . . . .

. . . . . .

[20 *verso.*] i. A mere fragment 12,000 *mūḍhiās* ... .

ii. *Example.*—A snake eighteen *hastas* long enters its hole at the rate of one half *plus* one-ninth of that *minus* one-twenty-first part of an *aṅgula* a day. In what time will it have completely entered its hole

$(\frac{1}{2}+\frac{1}{18}-\frac{1}{21})$ *aṁ* : $\frac{1}{360}$ years : : 18×24 *aṁ*° : 2 years 4 months 1 day.

iii. *Example.*—A worm... ..(see Mahāvīra. V 5).

M 2.

33 verso.

udā° || sumeru pṛithivi śamku surānāṁ parimāśrayaṁ ||

āga . χ kaśchi tarasā suramadiraṁ ||

satataṁ sapta-sārdhāṇāṁ sa pamadhya . . . ||

sa tṛi-bhāgā tṛi-pamchāṁśa nityam evaṁ cha gachchhati |

yojanānāṁ sahasrāṇichatur-āśitir uchchhṛitam |

kena kālena sau gachchhe vada me ta śuniśchitaṁ ||

. . .	7	dī°	1	yo	84000	.	adha chchhedaṁ 360* di
	1		1		1		
	2						

. . . . . .

M 2. [33 *verso.*] *Example.*—From the home of the gods a certain person desires to ascend swiftly SUMERU, the pole of the Earth and the dwelling place of the gods. He goes constantly at the rate of seven times one and a half and its quarter with one-third and one-fifth. The height of Sumeru is eighty-four thousand *yojanas*. In what time will he reach the summit? Give me well considered answer.

There is some doubt about the rate of going and the only clear parts of the statement are the second and third terms (1 day and 84,000 yojanas), but possibly the complete statement was

$7\,(1\ \tfrac{1}{2})\,(1+\tfrac{1}{3})\,(\tfrac{1}{3}+\tfrac{1}{5})$ *yo* . 1 day :: 84,000 *yo* . $\frac{120000}{16}$ years = $33\frac{1}{3}$ years.

33 recto.

i. udā° || dināra ko nāma viśā . . tṛi du χ khārjanīyaṁ sukha-bhojane cha |

tasyārdham ardhaṁ cha yad ardham ardhaṁ ta ke . *deva* guru prasādaṁ

kṛipaṇa dhana bhuktaṁ ||

1	1	1	1	1	1	1	108	pha°	dī°	1	dhā	8	d
1	2	2	2	2	2	2	1						

ii. uda° || ardhaṁ . . stāraṁ nava roma śatāni cha |

dvādaśa stīti charmāni kati romā . .

		12	24					
		1	1					
900	12	24			12	24	pha°	roma
	1	1			1	1		

[33 *recto.*] i. *Example.*—The earning of *dināras* is difficult but consuming them is easy. One gives one-half increased by ration of one-half (six times) for food for the poor. What is the amount consumed in 108 days?

$1 : \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} :: 108 : 1\ dī^{c}\ 8\ dhā^{c}\ 1\ am^{a}$

<i.e., $\frac{108}{64} = 1\frac{44}{64} = 1 + \frac{5}{16} + \frac{1}{16}$ and $4\ am^{c} = 1\ dhā^{c}$ and $12\ dhā^{c} = 1\ dī^{c}$>.

See Part I, § 110.

ii. *Example.*—(This is not understood, but appears to refer to the number of hairs on the skin of an animal.)

M 3.

32 recto.

. chandraanibhāṇa

. . . . . tu gaganaṁ nīta rāvaṇe | ra . . . . . yaṁ

tyakta sutaya śetayā | sā kai kena parāvartaṁ dhanur bhāga śa

pa . . . vane patamānasau daśa bhāgaṁ nidhāryate | evaṁ tat

parimāṇa . hīya mānaṁ tu nityaśaḥ kiyatas tu parāvartaī bhūmiṁ

prāpyayate ja .

dha° 1 1 / 1 10 + / 5	parā° 8	yoja° 30 / 1	chhe° 8000 / 1	yo°-ja °

phalaṁ parā° 218181 śe 9 / 11

ii. udā° || nāga śva chchharma gāmi *dratama daśa*

. . . . . .

M 3. [32 *recto.*] Folios 32 and 36 have the same knot.

i. A mutilated example about RAVANA and (?) SITA. When Sitā had been carried up 30 *yojanas* into the air she dropped something to earth, which turned over 8 times in $1\frac{1}{10}$ *dhanus*. How many revolutions did it make before reaching the earth ?

Solution.—$(1\frac{1}{4}-\frac{1}{10})$ *dha* : 8 revolutions : : $30\times8{,}000$ *dhanus* : : 218,181 $\frac{9}{11}$ revolutions. (There is a fair amount of conjecture here. See Part I, § 47).

ii. *Example* (?).—A snake which is 100 *yojanas*, 6 *krośas*, 3 *hastas* and 5 *angulas* long sheds its skin at the rate of 1 *angula* in 2 days. In what time will it be free ?

(The solution is given (?) on the reverse.)

32 verso.

i. | 100 / 6 / 8* / 3 / 4000* / 5 / 24* | ūrdha chchhe° 768000 a°-yo° / 1 | 1 adha chchhedaṁ 768000 phalaṁ

va° 429867 mā° 1 di° 4

ii. udā° || vraja . . charīśvāktā patitaṁ bhūmi tale paṭaṁ |

tṛi-śatāṁsya . . nāṁ tu saptа yojana hīyate |

chatur daśas tu koṭṭī . *h*ūyata paṁcha-śashṭi cha |

kai dinai bhūtale prāpya va*da me ganakottama* ||

nyāsa sthāponaṁ kṛiyate |

[32 *verso.*] i. 1 *aṅ*°. 2 *d* : : 100 *yo*° + 6 *kro* + 3 *ha*° + 5 *aṅ* : 429,867 years 1 month and 4 days.

<or 1 : 2 : : 77,376,077 *aṅ*° : $\frac{144{,}753{,}184}{360}$ years>. See Part I, § 108, for the measures employed.

ii. An example about some garment falling to the earth. The elements are uncertain. Compare with the problem on the obverse (i).

M 4.

i. hyā pañcha triguṇita sakhē — 36 recto.

. . . . . esha deśa pramāṇaṁ samāptaṁ ||

ii. udā || sa . . lavanasya rāshe koshṭhatāṁ va kṛitāṁ rharai |

eshāṁ chaikāṁ rāśi punar e . . . . dhā nītā |

saptāṇāṁ m api chaikā rāśis tulitāṇi |

pañcha saptatyā . . sahasraṁ bhavet saptāshṭa guṇaṁ kim

rā 1	1075	56	adha chchhedaṁ 2000* pa°-bhā°	pha- bhā 30
1	1	1		pa° 200

esha rāśi lavaṇa pramāṇaṁ

iii. kākiṇī daśa bhāgasya dadyād ashṭādaśīti . . |

tasyāṁ viṁśati bhāgas cha śata bhāgaṁ prayachchhati |

naro vakshaśa . . . . . .

M 4. [36 *recto.*] i. 'This land measurement is completed' may refer to the fragmentary example at the bottom of folio 32 *verso*, but I doubt it.

ii. The example appears to refer to heaps of salt. If one heap or quantity weighs 1,075 *palas* how much will 56 heaps weigh ?

1 : 1075 :: 56 : 30 *bhā* +200 *pa*°

∴ or $\frac{1075 \cdot 56}{2000} = \frac{60175 \cdot 0}{1000} = 30$ *bhāra* = 30 *bhā* +200 *pa*° >.

iii. One tenth of a cowry is given in eighty-eight. . . Of this one-twentieth and one hundredth.....

i. | va° 3 | 1 | 1* | yo° 5 chhe° 4608000* ya°-yo° | pha° va° 21333 ma° 4 | — 36 verso.
| 1 | 1 | 360 | 1 | |

ii. yojanasya tṛibhāgārdhaṁ sa tṛibhāga padonakaṁ |

yā nau dinat tṛibhāgena . . . gena gachchhati |

śā puna ϕ pañcha bhāgārdhaṁ yojanasya tathāshṭamam

. . . ti nivartaṁte vāyu vega valāhatā |

yojanānāmshṭau tara śataṁ *kena* kālena gachchhati ||

di 1 bhā 1	1 gu 1	1 1	3 bhā
3 3	2 5	2+ 8	

[36 *verso.*] i. The statement means 3 *ya*° : 1 day : : 5 *yo*° : : 21,333 years 4 months or 3 *ya*° : $\frac{1}{360}$ years : : 5 × 4,608,000 *ya*° : 21,333 years 4 months < $\frac{5 \times 4,608,000}{5 \times 360}$ = 21,333⅓ >. For the measures see Part I, § 108.

ii. A boat travels ½ of ½ + ½ — ½ *yojanas* in ½ of a day, but is driven back by the wind ½ of ¼ of a *yojana* in ½ of 3 days. In what time will it travel 108 *yojanas* ?

$\frac{108}{(\frac{1}{2}\text{ of }\frac{1}{2})+3-(\frac{1}{2}\text{ of }\frac{1}{4})+1}$ = < 1 year, 3 months, 12½ days >.

The problem is something like this but the details are not clear and the lower part of the statement has disappeared. See Part I, p. 51.

M 5.

34 recto

i. *khagā ekādaśā* bhuktā prasṛitiṁ chaiva meva cha ‖

. . shṭau vada sakhe kiṁ khagaṁ vada sundari ‖

pra° 1	kha° 11	khā 5760	phalaṁ khaga* 63360.
1	1	1	

esha bāhu pramāṇaṁ ‖

ii. kaśchit pumāṁ suvarṇas tu· kalā pāda yutaṁ yavaṁ ‖

pratyahaṁ sūline śuddhi · . . . kila dattavāṁ |

pamchābdai māśam evaṁ tu dinai paṁchadaśas tathāḥ

datvā . . sya sarvāya jñātum ichchhāmi tatvata ‖

di 1	1	6* bhā	5	chchhedaṁ 192* yava-tola
1	1	1		
	1	4		
	4*			

M 5. [34 *recto.*] i. The problem is: 'Eleven birds feed on a *prasṛiti* (handful) of corn, how many can feed on 8 Kharis of corn? It ends "Say, O friend, say what are the *Khagas*, O SUNDARI."

If this is correct, the name Sundari, 'beautiful one,' is used in exactly the same way as Lilāvati is used by Bhāskara.

The solution is 1 *pra* : 11 *kha* :: 8 *khā* : 63,360 *khagas* which would make 720 *prasṛis* = 1 *khāri*: but there are many elements of doubt and the application of *esha bāhu pramāṇam* to this particular problem is not clear.

ii. By certain persons one *kalā* plus one *pada* and one *yava* are given in gold daily at the shrine of ŚULIN. What would be the amount of the gift in five years, five months and fifteen days.....I desire to know that

Solution.—1 day: 1 *ma* + 1 *ka* + 1 *pa* :: 5 y. 5 m. 15 d. : x :: or 1 d. : 30 *pa* :: 1,965 d. : $\frac{1965 \times 30}{192 \times 2^0}$ *tola* = 12 *to* +3 *dhā* +1¼ *ama*>

See Part I, § 111.

34 verso

i. . . . . .

chittṛitāṅgai | tānī yata *śara-paramparay* ārjunena gṛiddhra . . . . tayā

spṛiśanti 1	śa° 1	yoja° 777 1 8	phalaṁ . . . 940
8	1	222 7 chchhe	
1			
9			
1			
5			

ii. māśakārdha yuto dhyanta . . . vista paṁchapaṁchāśa satereṇa vajra maṇai labdham . . tra kathayaśva mūlyaṁ śāṇa chaturbhāgasya siddhārtha paṁcha bhāgasya.

ku° 1	chhe° 128*	mā°-ku° 1	mā° chhe 40*	si°-mā°	sa° 55
1	1	2			1
2					

[34 *verso.*] i. The fragment *chittritāṅgai* .... *arjunena gṛiddhra* is extremely interesting although it throws no light on the problem. See Part I, § 47.

The statement is puzzling: it may mean

$\frac{1}{8} + \frac{1}{8.9} + \frac{1}{9.9.5}$: 1 *sa* :: 777 *yo*° + 222½ *kra*° : 40

But all the terms except the second are ambiguous.

ii. The problem is about a diamond weighing 1¼ *māshaka*, and obtained for ? 55 *satera*.

The statement means 1¼ *ku*° + ¼ *mā*° ; 55 *sa*°, and indicates that 128 *mā*° = 1 *ku*° and that 40 *si*° = 1 *mā*°. See Part I, § 111.

The whole page is an interesting puzzle. (Is the leaf a double one? Neither side shows any clear lenticle.)

M 6.

i. . . . . sūrya māṇasya

37 recto.

divākarasya ghaṭikaiχ kim prayatasya vada . niśchitaṁ

30	mu^{c}	chhe^{c}	2*	gha^{c}	-mu^{c}	500,000,000	gha^{c} 1 pha^{c} yo^{c} 83·333333$\frac{1}{3}$
1			1			1	1

ii. bhāṇo ratha(ṁ) sura mahoraga siddhasaṁ(g)hai vidyādharai ϕ parivritaṁ . . .

ahorātru | koṭī śatārdhaṁ sa rathaṁ pryāsyat tad bruhi śastra

kuśalo . . . vaktuṁ || muhūrtam ekena kiṁ gachchhe brūhi me

ganakottamā ||

500000000	gha^{c} 2	pha^{c} yo^{c} 166,666,666$\frac{2}{3}$		

M 6. [37 *recto.*] i. The question may be roughly restored: The Sun (*sūrya*) traverses 500,000,000 *yojanas* in a day. State with certainty the amount of the journey of the sun (DIVAKARA) in a *ghaṭikā*

The statement means 30 *mu^{c}* : 500,000,000 . 1 *gha^{c}* : 83,333,333$\frac{1}{3}$ *yo^{c}* and it indicates that 2 *ghaṭika*=1 *muhūrta* ($=\frac{1}{30}$ of a day). The origin of the length of the daily journey of the sun, namely 500,000,000 *yojanas*, is not known. See Part I, § 100.

ii. The chariot of the sun (BHANU) is surrounded by the groups of gods, great snakes, SIDDHAS and VIDYĀDHARAS. In a day and night its journey is said to be half a hundred *koṭis*. Tell me, O best of calculators, how much in one *muhūrta*?

30 *mu^{c}* : 500,000,000 :: 2 *gha^{c}* 16,666,666$\frac{2}{3}$ *yo^{c}*

i. . . . . bhage bhaved rāśi |

37 verso.

ūrdha chhhedaṁ 108000 viliptāṇam . . . . liptā 5

ii. paṁchārdha saṁvatsare bhukte rāśaikā yadi bhānujaḥ brūhi . . . ka tatvajña

samaśve vāsareṇa kiṁ

2	rā^{c}	1		1	aṁ^{c}	1	
1		1		1		360	
2							

ūrdha chhhedaṁ 108000 viliptāṇāṁ rāśi | adha chchhedaṁ $\frac{1}{6}$ viliptā lipta ||

phalaṁ viliptā 2 || esha graha gatiṁ ||

iii. udā° || rāja yudhiṣṭhiro nāma ϕ pāṇḍu-vaṁśa . . . .

[37 *verso.*] i. The remnant of a problem possibly related to the daily motion of Jupiter, which according to the *Sūrya Siddhānta*, amounted to very nearly 5 minutes of arc (*liptā*).

ii. If BRIHUJA (Saturn) moves through a sign in two and a half years, state, O knower of the truth, what will its motion in a solar day be equal to.

The solution is $2\frac{1}{2}$ years : 1 sign :: 1 degree : x

and $x=\frac{1\text{ sign}\times\text{sin degree}}{2\frac{1}{2}\text{ years}}=\frac{30\times60\times60\times2}{5\times360\times360}=\frac{108,000}{900}=120''=2$ minutes of arc (not 2 seconds as stated in the text, where *viliptā* appears to have been written by mistake for *liptā*). The terms employed are all orthodox except perhaps *vāsara* for 'solar day', but its special use is quite intelligible.

See Part I, § 100; and also my *Hindu Astronomy*, p. 57.

iii. This fragment is of interest because of the reference to YUDHISTHIRA. See Part I, § 48.

M 7.

i. . . . . vyūha pārthaṁ hehayakī . . . . ghnata 47 verso.

sāyakaiś chaiva ϕ patti sva-pāda dala śodaśai |

a . . nyā chatasrā vai hatā tena mahātma vāṁ ||

śarāṇāṁ cha parīmāṇam . . . . viśārada ||

śi 1 / 1	16 / 1 / 1 / 4 / 1 / 2	4 a° chhe° 21870 / 1 / 1	phalaṁ śarā 2624400

anyā . . . . . 1 pramāṇaṁ

ii. sūtraṁ || eko ratho gaja . . . . . .

M 7. [47 *recto.*] i. This appears to relate to PARTHA the Mahābhārata hero, who pierced each soldier with 16 $(1+\frac{1}{2})$ $(1+\frac{1}{2})$ arrows and slew four divisions of the army. How many arrows did he use?

1 *śi°* : 16 $(1+\frac{1}{2})$ $(1+\frac{1}{2})$ :: 4 × 21,870 : 2,624,400.

The abbreviation *śi°*=? ; *a°*=*anīkinī*. See Part I, § 52.

There is a very similar example about Pārtha in the *Līlāvatī* (§ 67) which has already been quoted (Part I, § 47).

ii. *Rule.*—There is little doubt that this rule relates to the constitution of an army and is exemplified on the reverse (fol. 47 *recto.*)

i. . . . . 47 recto.

. . . . . vicȟakshaṇaḥ

chamūs tu pṛitanās tisras tisraś cha . . . .

anīkīni daśaguṇām āhu arakshohanī buddaḥ ||

[47 *recto.*] i. Apparently 3 *chamūs*=1 *pṛitanā*, 3 *pṛitanās*=1 *anīkinī* and 10 *anīkinīs*=1 *akshauhiṇī*. The statement mean: a *patti* consists of 1 *ratha*+1 *gaja*+5 *nara*+3 *turaga* (i.e., 1 chariot+1 elephant+5 foot soldiers+3 horsemen) and that an *akshauhiṇī* contains 3.^{2}10 of each of these, namely—

3.^{2}10.1 chariots	=21,870 chariots.
3.^{2}10.1 elephants	=21,870 elephants.
3.^{2}10.5 foot-men	=109,350 foot-men.
3.^{2}10.3 horsemen	=65,610 horsemen.
TOTAL	=218,700.

Albirūnī (Chap. xlviii) gives the following scheme:—

Each akshauhiṇī has 10 anīkinī.
" anīkinī " 3 chamū.
" chamū " 3 pṛitanā.
" pṛitanā " 3 vāhinī.
" vāhinī " 3 gaṇa.
" gaṇa " 3 gulma.
" gulma " 3 senāmukha.
" senāmukha " 3 patti.
" patti " 1 ratha

and "a *ratha* comprehends besides, one elephant, three riders and five footmen."

Possibly all these terms were included in the example but *vāhinī*, *gaṇa*, *gulma* and *senāmukha* are now missing. Numerically Albirūnī's scheme is identical with that given in our text.

The abbreviation *tu°* in the text is probably for *turaga* 'a horse.'

See Part I, §§ 51 and 94.

M 7—*contd.*

akshohi . . .

ra° 1	esha	3	3	3	3	3	3	3	10	gu^{c}
ga° 1	pati	1	1	1	1	1	1	1	1	
na° 5	guṇitā jātā							ratha	21870	
tu° 3								gaja	21870	
								nara	109350	
								haya	65610	
									(218700)	

esha akshohiṇī pramāṇaṁ ||

ii. udā° || kaśchid rāja kumāra śatrudama | . . . . . .

ii. *Example.*—A certain prince SATRUDAMA [The phrase may as well mean: 'a certain prince (engaged in) curbing (his) enemies, (employed or fought so many soldiers)—K. N. D.]

M 8.

. . . ki	di° .	ra°	1	. . . . .					*va*°	3	. chhe . .	48 recto.
		ya°	1	3*	bhā°	ksha-	80*			1		
				1		-ya	1			1		
				5			3		mā°	3		
		ka°	1	6*	bhā°					1		
				1						12*		
				4								
		pā°	1						di°	1		
			4*							30*		
		śe°	1									
			3									

*chhe*daṁ 480* rakti-pala . . . guṇitaṁ jātaṁ 419942 . . . . . 36 pala
115200

to° 8* pale-to° 3 tolen āsti dhā° 12* dhā° 7 dhāṇe nāsti aṁ° 4* aṁ° 2 .

i 8. [48 *recto.*] This is a statement belonging to some lost problem and, omitting the change-ratios (marked with asterisks), it means
5 days : 1 *ra*°+1 *ya*°+1 *ka*°+1 *pā*+½—½ ? :: ? years +½ month+1 day : 36 *pa*°+3 *to*°+7 *dhā*°+2 *aṁ*° .....
or 5 : 110½ *pā*°—... :: ? years, etc. : 36 *pa*+3 *to*+7 *dhā*+2 *aṁ*°...
or 5 : $\frac{231--}{115200}$ *palas* :: ? : $\frac{419942}{115200}$.
(Therefore the third term must be of the order $\frac{5\times419942}{115200}\times\frac{115200}{235\times365}$ or nearly 180 years.) The abbreviations employed, the change-ratios, and the measures are explained in Part I, §§ 103 and 111.)

M 8--*contd.*

i. ... phalaṁ bhā° 2 . . . . . **enāsti** 48 verso.

pala 2000 bhā° | pa° 270 || . . . . . **to° 8**

chhe'	8*	
dhā°	2	
chhe°	12*	
guṁ°	3	
chhe'	5*	
ya°	2	3* bhā
	1	1
		5

tola-pala to' 6 tole nāsti dhāṇe 12 dhā° 8 ||

ii. yadi dinaṁ ekena esha dattaṁ tad dvādaśa varsheṇa

di	1	216	bhā°	varshe 12	3	. phalaṁ bhāra 93 . . .
	1	270	pa		1 1	
		2000*	chhe'			
		6	to'			
		8*	chhe°			
		8	dhā'			
		12*	chhe			

M 8. [48 *verso.*] This exhibits two mutilated statements of proportion that evidently belong to the same problem
i. The first is ? : ? + 2 *dhā'* + 3 *guṁ°* + 2 *ya'* ? 216 *bhā* + 270 *pa'* + 6 *to'* + 8 *dhā* + ...
ii. If this is given in one day what is that in twelve years. . 1 day 216 *bhā* + 270 *pa'* + 6 *to'* + 8 *dhā* + ... : 12 years + ... : 93..... *bhāra* or < 1 · 216 *bhāras* (nearly) · 4320 days (nearly) · x, and x · 216 · 4320 · 933700 *bhara* (roughly) >.

M 9.

. . . . rakti kshaya . . . . pañcha guṇaṁ . . 49 verso.

divasā viṁśatikaṁ kiṁ *śuṁdya*ti mah vada niśchayaṁ

1	to	3		kshaya 4 +		va' 25	chhe' 360	
1	mā°	2			60*	ma' 5		1
		12*		śi' 4				
	am'	3			8*	di° 20		
		4*				30*		
	ya'	3						
		4*						
	ka°	1	6*					
	pā°	1	1					
		4*	4					
	mū°	1						
		4*						

62321 / 19200	kshayaṁ śodhya . .	60881 / 19200	adha chchhedaṁ 2000 . .
sarva guṇitam	558278770 / 19200	. .	7 tola palaṁ . . . . .

M 9. [49 *verso.*] The statement means (omitting the change-ratios which are marked with asterisks) 1 day : 3 *to°* + 2 *mā°* + 3 *am°* + 3 *ya°* + 1 *ka°* + 1 *pa'* + 1 *mū°* — (+ 4 *rn°* + 4 *śi°*) :: 25 years + 5 months + 20 days : x
or 1 day : $\frac{4321}{19200}$ *tolas* — $\frac{1443}{19200}$ *tolas* :: 9170 days : x, and x = $\frac{865,278,770}{19,200}$ *tolas* = *bhā°* + 1634 *pa°* + 5 *to°* + 0 *mā°* + 0 *am* + 3 *ya°* + 3 *ka°* + 3 *p°* 1 *mā*.

M 9—*contd.*

. . . . . 49 recto.

ya° 3 yavan*ā*sti ka° 6

1 . . . . ka° 4 kalānāsti pā

4

. . pādanāsti mudri° 4 *pāmu* mū° 2 ||

udāharaṇaṁ ||

. . . śūkhyair yajaṁti devī pratimahni kechit dadāmi devyā . . kaṁchaḥ kṛitvā dīnāra śatāni chatvārita dhānakā aṁḍikā raktikā yavā kalā pāda mūḍrikā cha | etad mūlyaṁ vada me tatra m . syā kiṁ

1 to 12*	mū 400
1 1	1

dhā 1 phalaṁ dī 50 dīnāra nāsti dhāṇe
aṁ 1
4* 12* dhānakā 10 dhāṇe nāsti āṁ 4*

ra 1 1* bhā
1
4
aṁ 1 ||

ya 1 3* bhā
1
5

ka 1 6* bhā
1
4

pā 1
4*

mū 1
4*

[49 *recto.*] i. This is the end of the answer to the problem on 49 *verso*. See Part I, §§ 101 (*iv*) and 111.

ii. *Example.*—The first part is too broken up to make out, but it appears to refer to a gift connected with an image of Devī and worship by Śākhyas. (cf. *Śākhara*, the name of a Śaiva sect). [It is possible to read *Mākhyair* for *Śākhyair*, in which case the chiefs of some clan or territory are intended. K. N. D.]

The statement (omitting change-ratios) means—

1 *te°* : cent 400 :: 1 *dhā°*+1 *aṁ°*+1 *ra°*+1 *ya°*+1 *ka°*+1 *pā°*+1 *mū°* : 50 *dī°* +10 *dhā°*+1 *aṁ°* <or 12 *dhā°* : 400 *dī°* :: $\frac{1}{100}$ *dhā°* : x and x=50$\frac{1}{2}$ *dīnāras*>= 50 *dī°*+10 *dhā°* +1 *aṁ°*.

L 2

M 10.

i. | to° 1 | va° 5 | to° 1 | pha° va° 6 . . . śe° | 9 | guṇitaṁ 55 recto.
1		1	dhā° 1		10
		3	12*	7227
			aṁ° 1	1200
			4*
			ra° 1 1* bhā°
			1
			4
			ya° 1 3* bhā°
			1
			5
			si° 1 2* bhā°
			1
			2
			ka° 1 2* bhā°
			1
			2
			pā° 1
			4*
			mū° 1
			4*

atha śaḍdrammako . . . jjarad, vidhānakais dramam śā . . . . viṁśati-

pālā hatai dhānakā | asyaiva skandha-

ii. | to° 1 | va° 5 | to° 1 dhā° 1 1* aṁ° 1 1* ra° 1 1* ya° 1 1*
| 1 | | 1 | 1 12 | 1 48 | 1 60 | 1 192
| | | 3 |

si° 1 1* ka° 1 1* pā° 1 1* mū° 1 1*
1 480 1 120 1 4800 1 19200 |

M 10. [55 *recto.*] Folio 55 is here misplaced: it should come before folio 49, which has the same knot as 44.

i. The first statement means—

1 *to°* : $5\frac{1}{2}$ years :: 1 *to°* + 1 *dhā°* + 1 *aṁ°* + 1 *ra°* + 1 *ya°* + 1 *si°* + 1 *ka°* + 1 *pa°* + 1 *mū°* : x, and $x = 5\frac{1}{2} \times \frac{11111}{10000} = \frac{11111}{1800} = 6\frac{1}{1800}$ years = 6 years, $\frac{1}{10}$ days. But the answer given appears to be 6 years..... $\frac{1}{10}$.

ii. This is the same proportion with the change-ratios given in cumulative form. See Part I, § 104.

. . . . . pañchatriṁ satam | 55 verso.

divardha tolakasya divardha māsakasya .

divardha chāṇḍikā divardha yavasya kiṁ mūlyaṁ ||

[55 *verso.*] If 1 *tola* cost thirty-five *drammas* what will be the price of one and a half *tolās*, one and a half *māshakas* and one and a half *chaṇḍikas* and one and a half *yavas*.

M 10—*contd.*

nyāsa

to° 1	35	1 to°
1	1	1
		2
		1 1* mā°
		1 6
		2
		1 1* aṁ°
		1 2
		2
		1 1* ya°
		1 2
		2

pha° draṁ° 58 śe°

31
128

punānyaṁ

to° 1	35	1	1	1*	1	1*	1	1*	phalaṁ 58 śe°	31
1	1	1	1	12	1	48	1	192		128
		2	2		2		2			

Statement.—(i) 1 *to*° : 35 :: $1\frac{1}{2}$ *to*° + $1\frac{1}{2}$ *mā*° + $1\frac{1}{2}$ *aṁ*° + $1\frac{1}{2}$ *ya*° : $58\frac{31}{128}$ *dram*° or <1 : 35 :: $319\frac{1}{2}/192$: $58\frac{31}{128}$>.

(ii) This is exactly the same proportion with cumulative change-ratios indicated. See Part 1, §§ 104, 105

M 11.

. . . . 44 verso.

nivi sapta-śatānāṁ ka*x* kālaṁ ārjana bhakshane ||

nyāsa sthāpanaṁ kṛiya*te*

ā° 1	*di*° 1	*bhā*°	bha° 8	di° 5	bhā° 1	pa° 32	bhā	śū° 2	36	bhā°	1	1
1	1		1	1	1	1		1 1	1		1	360
2	3			3				2 4				

bhāṇḍā 700
1

vyaya rāśi

223
144

āya rāśi

280
61

etat kāleṇa ārjana bhaksh

M 11. [44 *verso.*] . . . . the capital is seven hundred. What is the time of the consumption of the earnings.

The statement means—

Daily earning $\frac{1\frac{1}{4}}{1\frac{1}{2}}$; given for Bha(vānī) 8 in $5\frac{1}{2}$ days; given for *pa(ra-loka)* 1 in 32; given for Śū(lin) $\frac{2\frac{1}{4}}{4\times 36}$; $\frac{1}{360}$ years; reserve 700.

<The daily earning is $\frac{5}{6}=\frac{120}{144}$>. The expenditure quantity is $<\frac{8}{5\frac{1}{2}}+\frac{1}{32}+\frac{2\frac{1}{4}}{4\times 36}=>\frac{223}{144}$. <The daily loss is $\frac{223-162}{144}=\frac{61}{144}$ so 700 will last $\frac{700}{1}\div\frac{61}{144}$> $=\frac{280}{61}$ years and 'in this time the earning will be consumed.'

Then 1 day : $\frac{223}{144}$ :: $\frac{280}{61}\times 360$: $2559\frac{1}{61}$ and this is the (total) expenditure in $\frac{280}{61}$ = 4 years, 7 months, $2\frac{18}{61}$ days.

Then the income, $1\frac{1}{2}$ days : $1\frac{1}{4}$ :: $\frac{280}{61}\times 360$: $1859\frac{1}{61}$, and $2559\frac{1}{61}-1859\frac{1}{61}=700$.

M 11—*contd.*

di	1	223	280	ūrdha chchhedaṁ 360 phalaṁ . . 2559 śe $\frac{1}{144}$	esha
	1	144	61		

vyaye ||

. . . va° 4 mā° 7 *di*° 2 śe°				28	
				61	

atha āya	di°	1	1	280	. . .	
		1	1	61		
		2	3			

i. . . .	2559	di	1	———	223	esha vyaya pramāṇaṁ	44 recto.
	1		1		144		
	61						

ii. udā° || eka daśārdham utpati sa tṛibhāga dina dvayāt

pūjārthaṁ sa tṛibhāgaṁ cha trayodaśa . *ta*tāś chayet

sāshṭa bhāga dinā trīṇi vāsudevasya chārchayet

pādoṇa trayodaśāṇāṁ cha ashṭa sārdha dināni chet ||

brāhmaṇā bhojane dadyā paraloka hitārthinaḥ

sa tṛibhāgam . jjaraṁ sa paṁcha bhāga dinattrayet

pa°

ardhaṁ sārdhaṁ dine

[44 *recto*. i. Again $\frac{1}{144}$: $\frac{2559}{61}$ :: $\frac{1}{1}$: $\frac{223}{144}$. This is the expenditure measure. See Part I. § 96.

Example.—One produces ten and a half in two and one-third days. For the sake of religion he gives thirteen and one-third in three and] one-eighth days; he offers for VĀSUDEVA one quarter less than thirteen in eight and a half days. Desiring reward in a future world he gave to Brāhmans for food one and one-third in three and one-fifth days ..... two and a quarter in five days .....

M 12.

. . . . . ārayet 43 recto.

. . sārdha dvādaśam evā tra bhojanē madyam uttamet

sa tṛi bhāga trayastriṁśai dinaid vāṇijyakasya tu. |

bhāṇḍāre dvādaśa śata vajārāṇāṁ sthitāsya vai |

eshā vyayasamutpattau kaṯ kālaṁ brūhi paṇḍita ||

karaṇa-vidhānena dvādaśa śatasya bhāṇḍāre sti ta .

10	2	bha°	13	3	bhā°	13		8	bhā	1	3	bhā°	1	1	bhā°	1	5	bhā°	2		1	*bhā°*
1	1			1	1		1+	1			1	1		2	1		1	1	[1	1	1		
2	3			3	8		4	2			3	5			2			3	[4	4	4		

12	33	bhā	1	1	bhāṇḍā	1200	guṇitāni
1	1		1	360		1	
2	3						

---

43 12. [43 *recto*] and also twelve and a half in thirty-three and one third days for the best wine for the consumption of merchants. In the treasure house was stored twelve hundred . Say, O Pandit, how long can this expenditure continue.

The statement means :—

Daily income = $\frac{10\frac{1}{2}}{33\frac{1}{3}} = \frac{3}{10}$.

Daily expenditure = $\frac{120}{33} + \frac{13}{3} + \frac{11}{30} + \frac{1}{12} + \frac{1}{3} + \frac{2}{5} + \frac{12}{33} = \frac{37}{10}$.

∴ The daily loss is therefore $\frac{37}{10} - \frac{3}{10} = \frac{34}{10}$ and $\frac{1200}{\frac{34}{10}} \times \frac{1}{360} = \frac{300}{153}$ is the period.

. . . . . .

2	10	800	adha-chchhedaṁ 360 diva . . teṇa saha || ya-piṇḍaṁ 43 verso.
1	1	727	
3	2		

2982	. . adhunā vyaya piṇḍaṁ		di°	1	1807	800
486				1	240	727
727						

ūrdha-chchhedaṁ 360 *phalaṁ* diva	2982	puna	800	2982	1
	486		727	486	1
	727			727	

*adha-chchhe*daṁ 360 di . . phalaṁ pratidina . | 1807 / 240 | evaṁ sarva

trai-rāśikena | udā°

[43 *verso.*] *Proofs.*—$2\frac{1}{2} : 10\frac{1}{2} :: \frac{800}{727} \times 360 : 1782\frac{34}{727}$ the total amount earned and $1782\frac{34}{727} + 1200 = 2982\frac{34}{727}$.

Again $1 : \frac{1807}{240} :: \frac{800}{727} \times 360 : 2982\frac{34}{727}$; and lastly $\frac{800}{727} : 2982\frac{34}{727} :: \frac{1}{360} : \frac{1807}{240}$ the daily expenditure. Thus each item (can be tested) by the rule of three.

M 13.

42 recto.

ārdha yukte trayo-daśa sārdhaṁ bhavati

40 bhā°	160	13
1	1	1
		2

. . . . eshāṁ chchhedāṁ kṛitā jātā ekeṇa . .

. . . sārdha trayo-daśabhi kiṁ iti | 1 | 4 | 27 | pha° 54 eshām . .
| 1 | 1 | 2 |

. ekena labdha chatvārish shaḍbhi saṁpadyate kathaṁ | 1 . . . 4 |
| 1 |

. . eko *labhati* chatvāri *śaṇsardh*asya tu kiṁ bhavet . . . . . .

. . . . . .

M 13. [42 *recto.*] This contains portions of a solution that is not, at present, fully understood. The preliminary work is missing and then comes the following proportion 40 : 160 :: 13½ : 54, or cancelling by 40 we get 1 : 4 :: ⅓ : 54. The next part is missing but apparently was—

1 : 4 :: 6 : 24
1 : 4 :: 3 : 12
1 : 4 :: ⅔ : 18

. . . . . .

42 verso.

i. jātā 54 | śaḍbhi 24 | 12 | ardhā 18 | ekatraṁ 54 ||

e . . . trai-rāśika karaṇa pratyeka mūlya vidhi ||

ii. aparaṁ vakshyāmi | vim*śānām* diva . . . kiṁ prathame khandhakeśu yo bhilikhita | apāsya prashṇā vidhi | 20 1 1 / 1 1 3 / 2 | . . . guṇaye | guṇitā

jātā | 20 3 1 / 1 2 3 | chhedaṁ | 20 1 1 / 1 2 | bhāge . . . jātaṁ phalaṁ rū

10 || esha viṁsānāṁ diva . . *bha*vati | atra uparimāś khandhakasya esha guṇākāraṁ bhavati | . . . . . .

[42 *verso.*] i. A fragment: 24+12+(24+12)÷2=54..... This *sūtra* gives the three term solution with respect to one price.

ii. I shall instance another.....what is that which is written in the first term? The solution is a matter of intelligence. $20\times1\frac{1}{2}\times\frac{1}{3}=20\times\frac{3}{2}\times\frac{1}{3}=20\times\frac{1}{2}\times1=10$.....Now this is the calculation of the foremost term.

M 14.

50 verso.

i. dramme tra*pu*sa śataṁ labdhaṁ ardheṇa labhyate χ kati |

eka rāśis tu kalanā gaṇita prakṛiyā kuruḥ

1 dramme	phalaṁ 50
100 trapusā	
1	
2	

ii. aparaṁ uda[ṁ] || sārdha dvaye . yasardha divardhe labhyate χ kati

2
1
2
1
2
1
2
1
2

iii. sūtraṁ || ardhen opari saṁguṇya . . vardha krameṇa cha |

ardheṇa ūrdhaṁ guṇaye ma . . paṁcha saṁguṇe |

bhājaye labdha paṇyaṁ

M 14. [50 *verso.*] i. The solution is 1 *dramma* : 100 *trapusā* :: $\frac{1}{2}$ 50.
ii—iii. The problem is too mutilated to understand. The *sūtra* seems to apply to the problem, but it is not clear.

. . . . . vaśishta putra

50 recto.

sikasyārthe putra pautra upayogyaṁ bhavatuḥ

likhitaṁ Chchhajaka putra gaṇaka rāje brāhmaṇena |

sarveshāṁ-m-eva śāstrāṇāṁ gaṇitaṁ mūrdhni tishṭati |

ādyāvasāne saṁsāre utpaṁnna . . . mahat

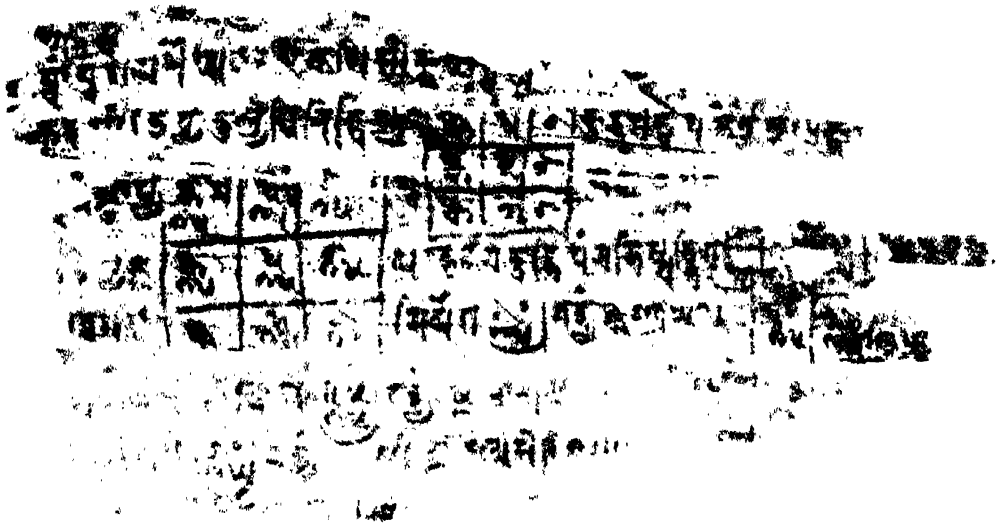
paśchā śṛishṭi tadā kartuṁ śivena paramātmana

. . yādyaṁ cha-m-utpaṁnnaṁ gaṇitaṁ sakhya kāraṇaṁ |

yach . . . . .

[50 *recto.*] At the top of this page is the remnant of a problem, too broken up to make out. The rest of the page is devoted to what appears to be a colophon. This is not all clear but what remains seems to state that the work was written by a certain Brāhmaṇa, a prince of calculators, the son of Chhajaka. It also refers to the importance of the science of calculation, which, it is said, we owe to Śiva.

MGIPC—L—III-9-6—2-6-33—550.

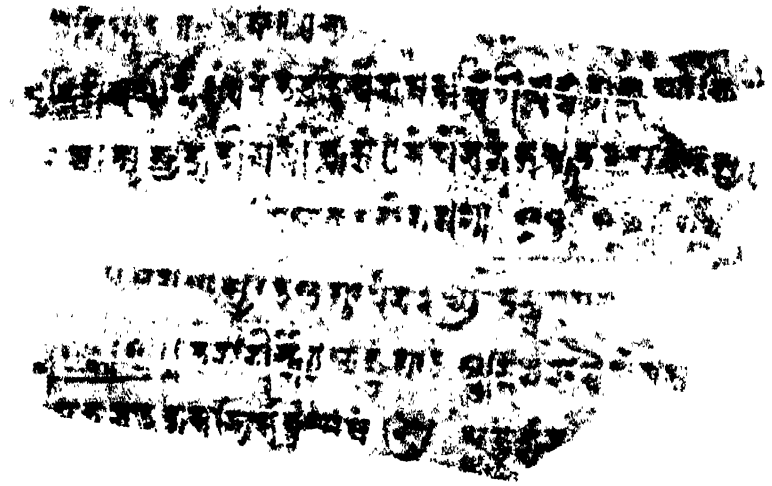


40 RECTO.E

40 RECTO.D

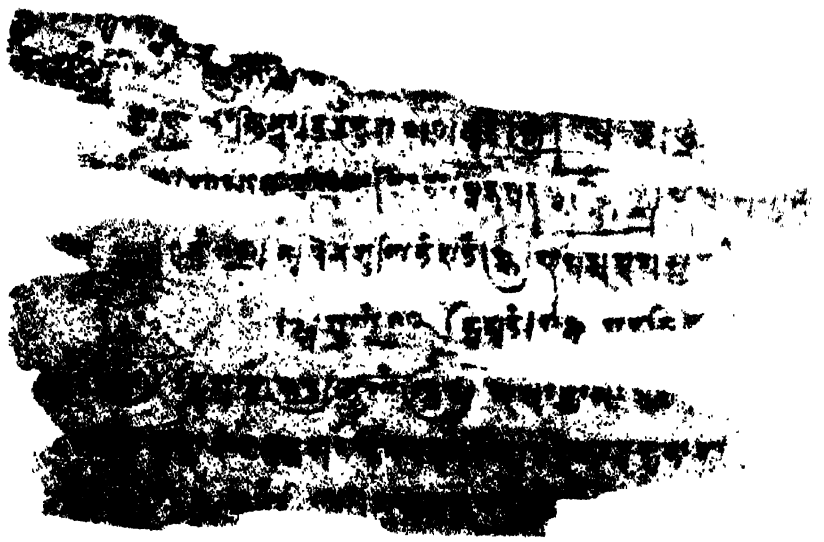
39 RECTO.E

38 RECTO A



39 RECTO.D

39 RECTO.B



23 RECTO.C

51 VERSO B

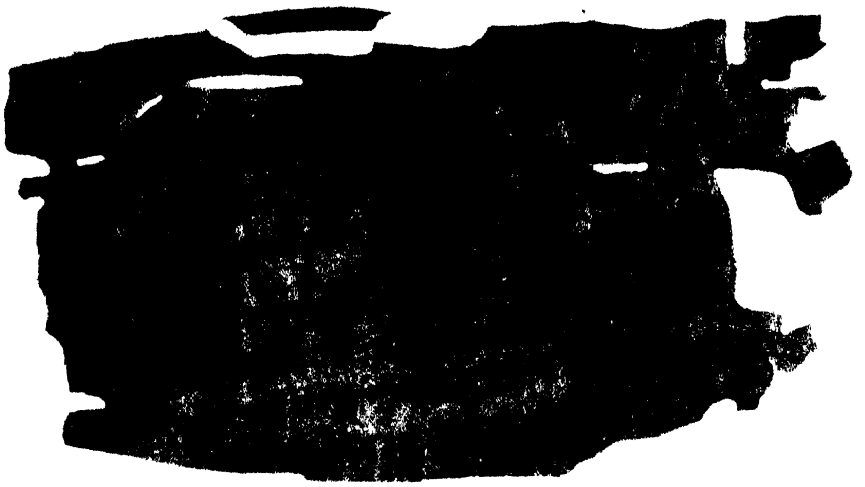
35 RECTO A

Photogravure

Survey of India Offices Calcutta 1923

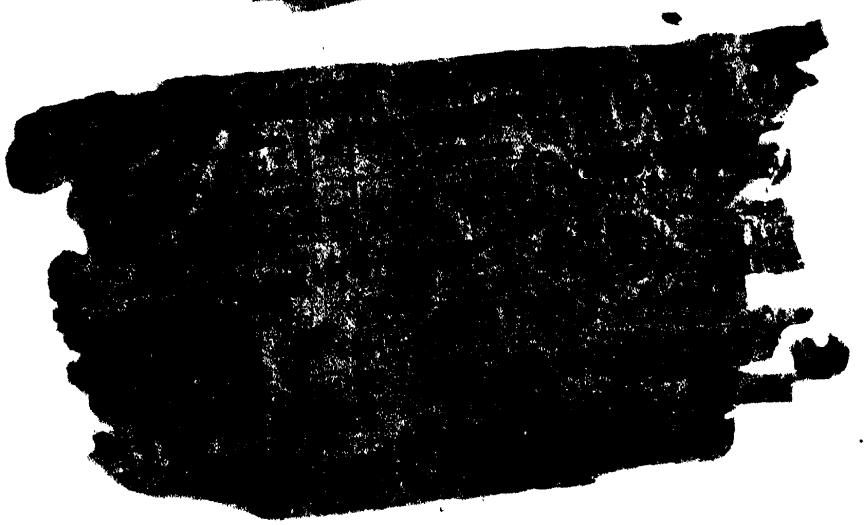
RE-ARRANGED FRAGMENTS

Plate IV



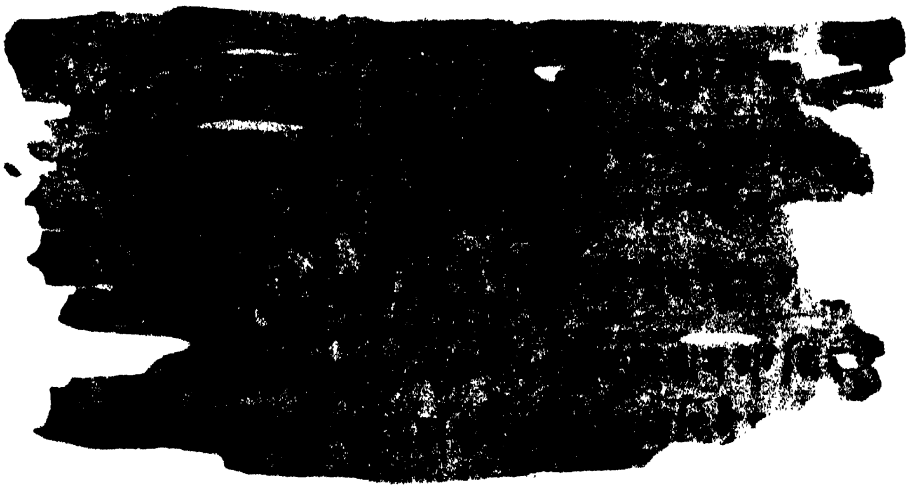
4 RECTO

4 VERSO



5 RECTO

5 VERSO



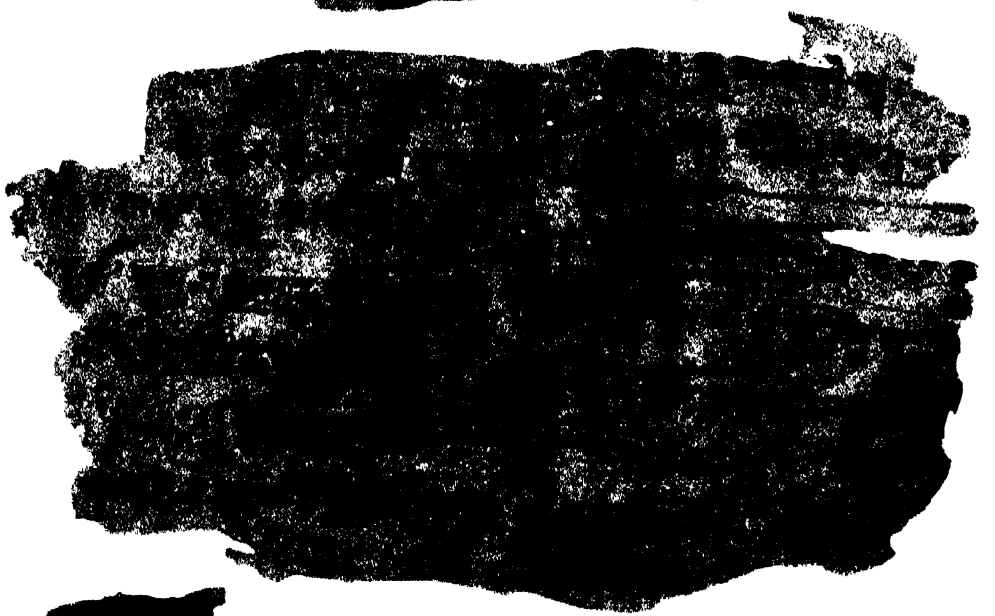
6 RECTO

6 VERSO

Plate VIII

10 RECTO

10 VERSO



11 RECTO

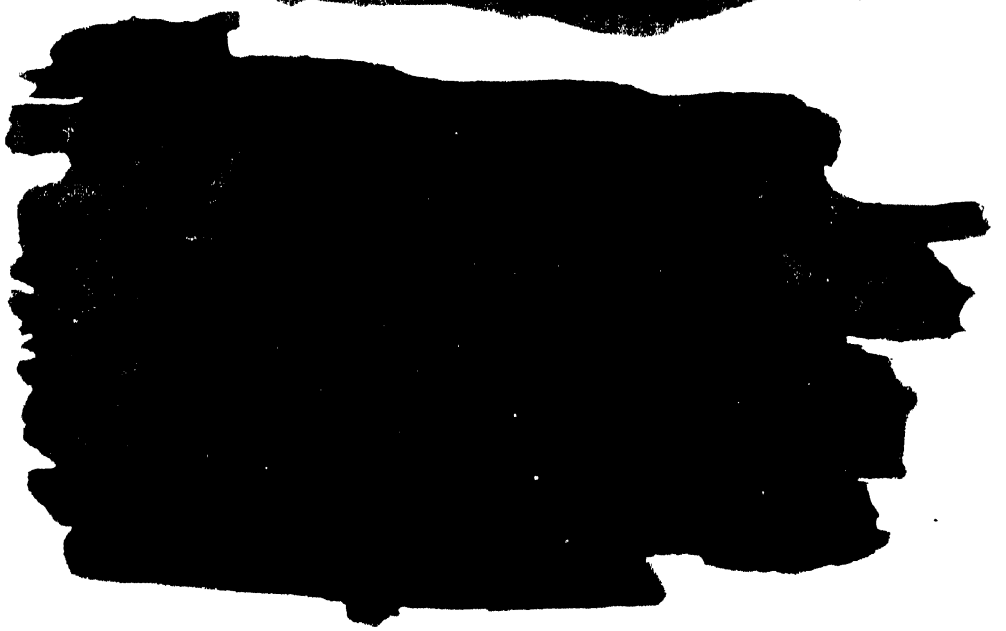
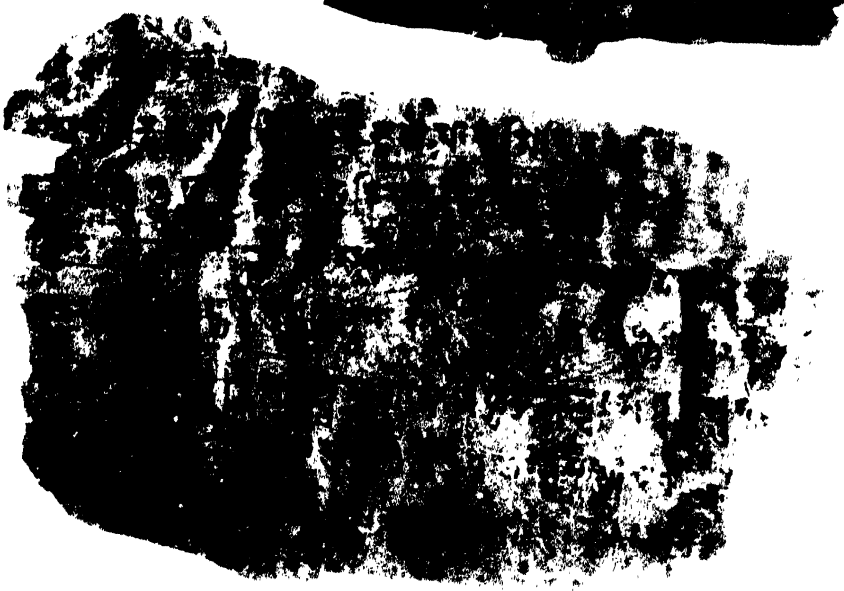
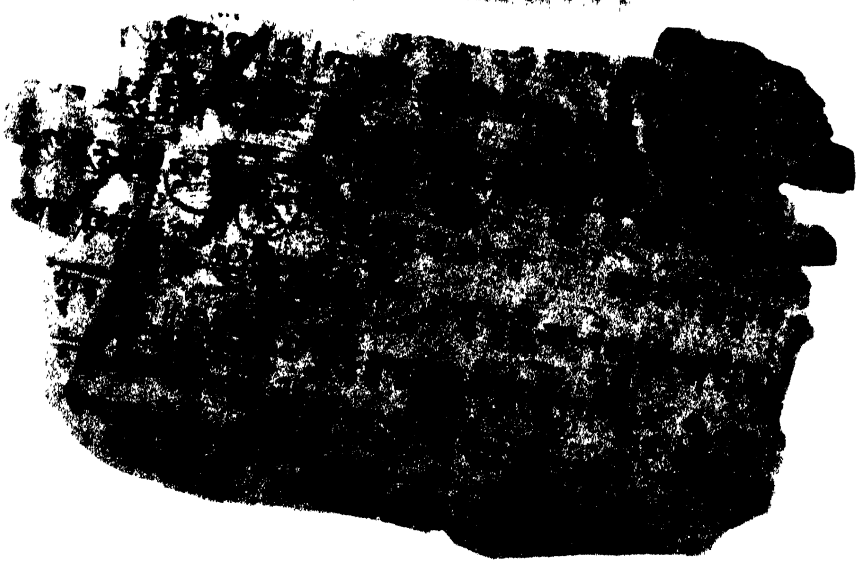


Plate IX

11 VERSO



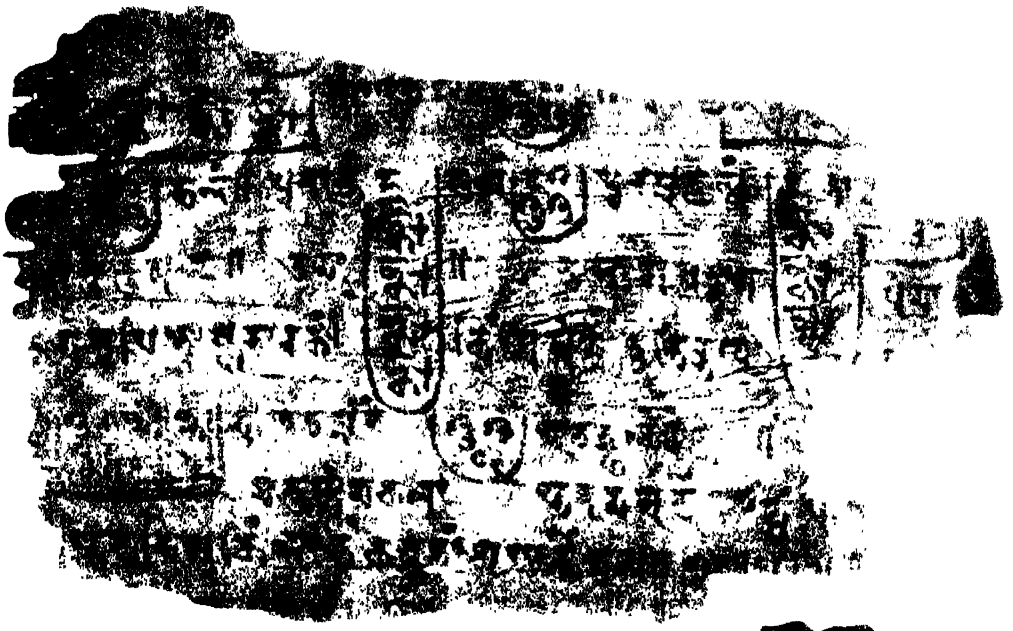
12 RECTO



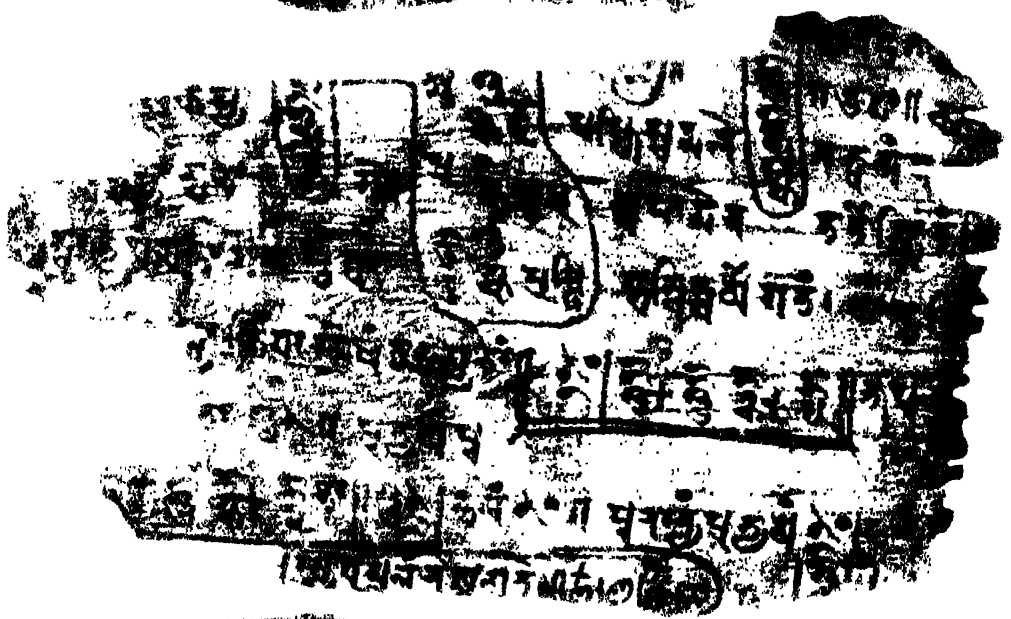
12 VERSO

Plate 3.

13 RECTO



13VERSO



14 RECTO

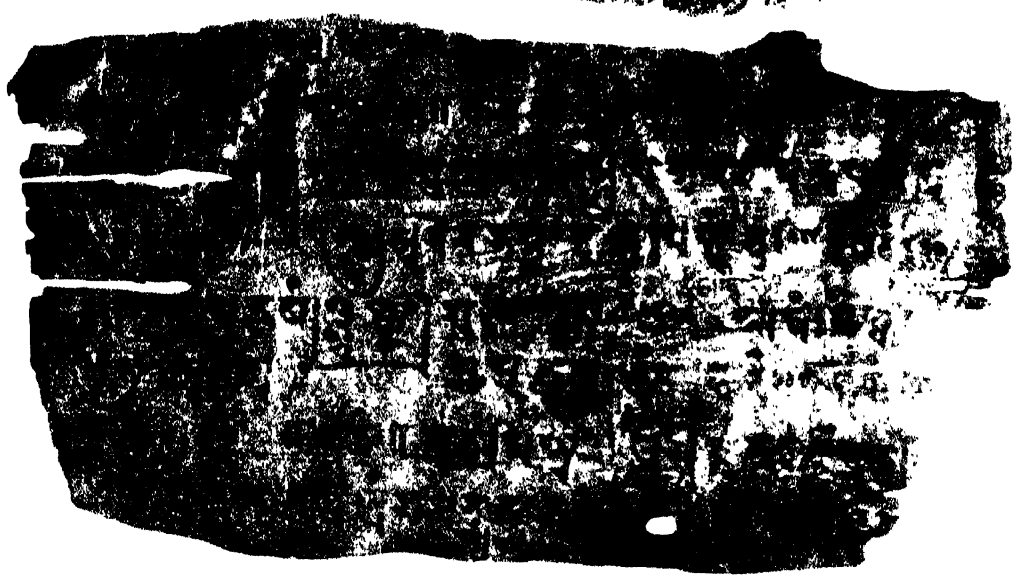
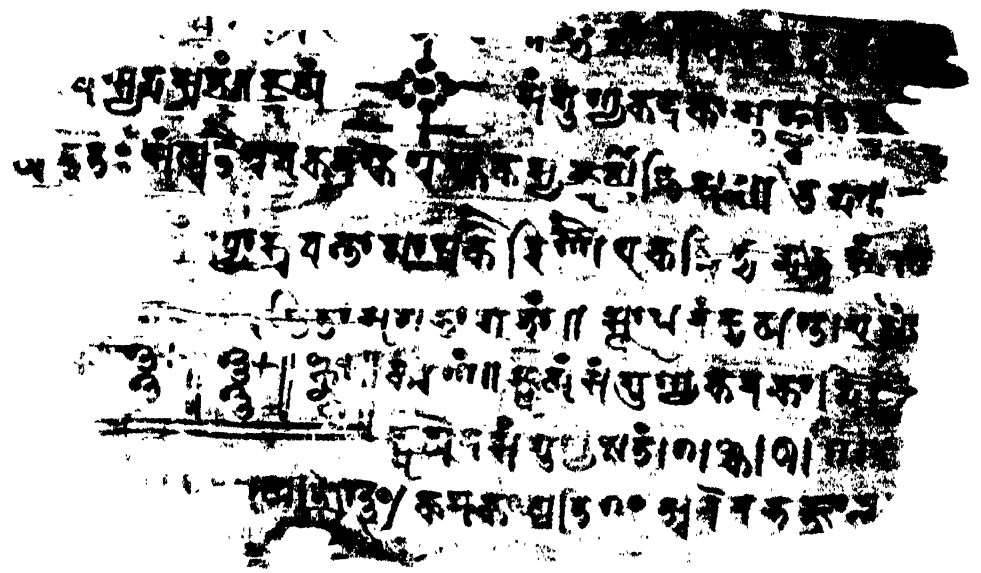


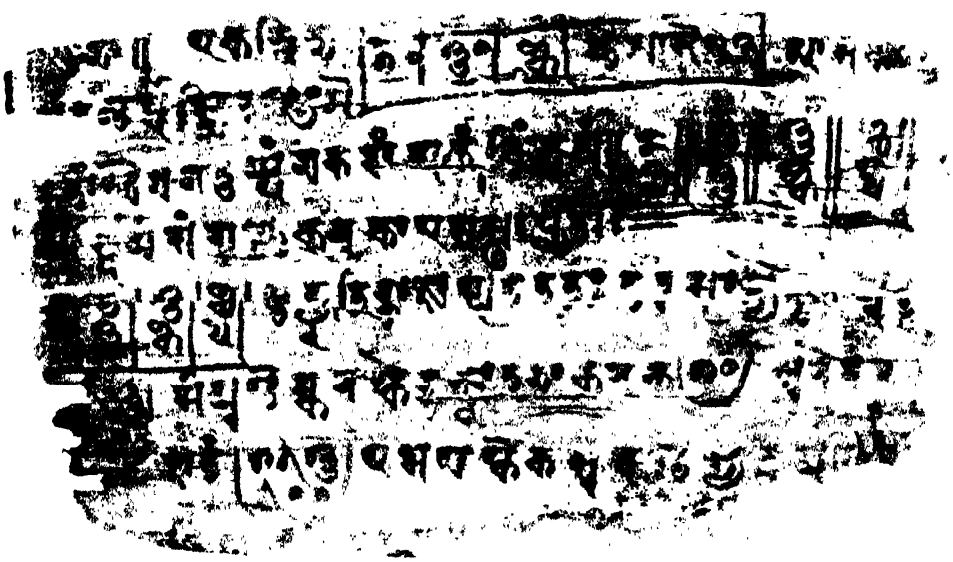
Plate XII

16 RECTO

16 VERSO



17 RECTO

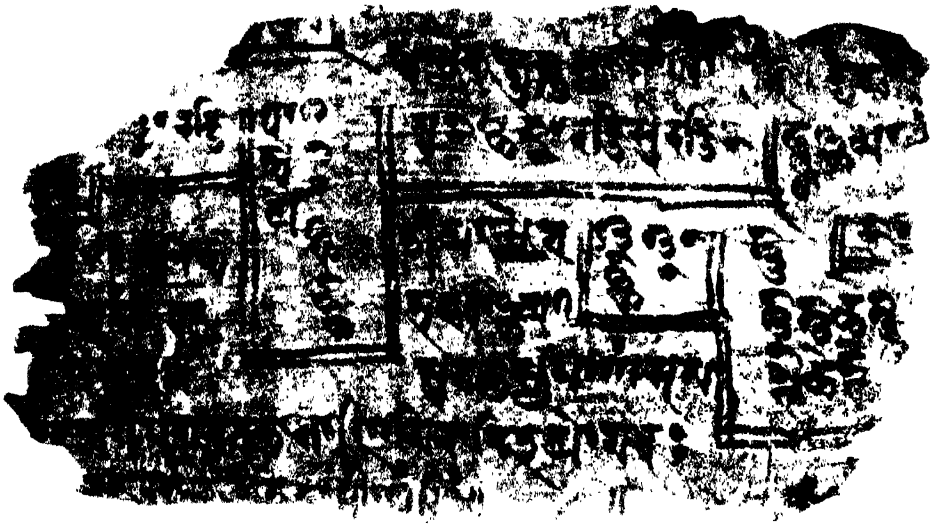


17 VERSO

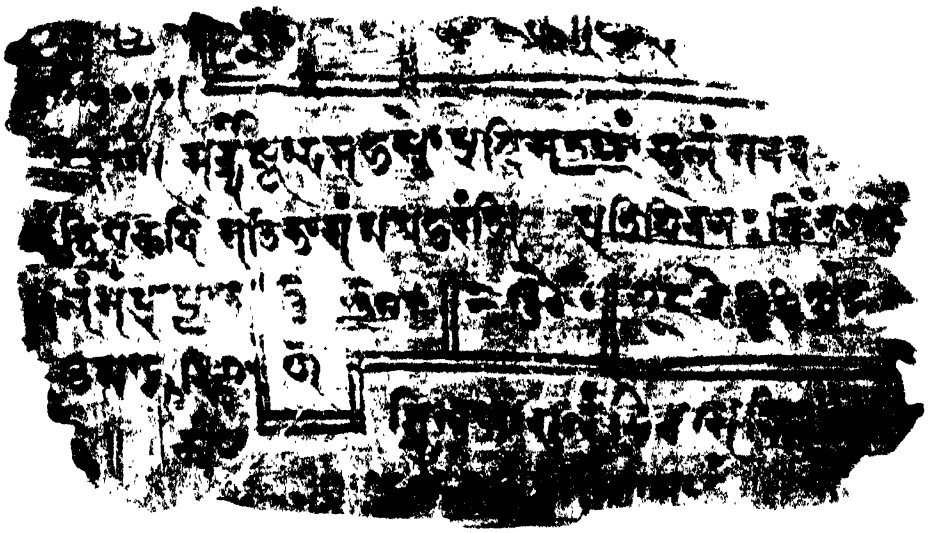
18 RECTO

18 VERSO

Plate XIV



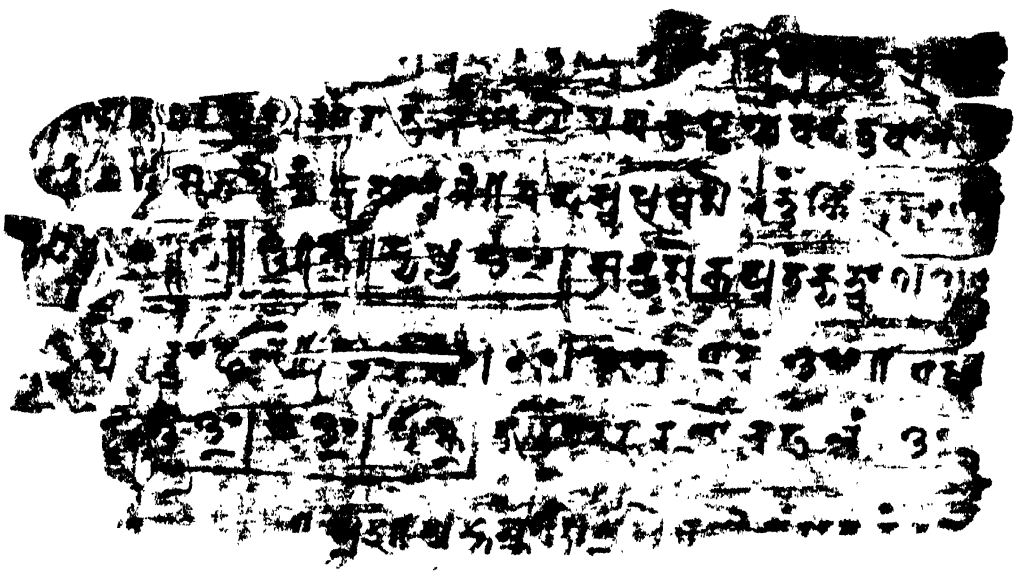
20 VERSO



21 RECTO

22 VERSO

23 RECTO

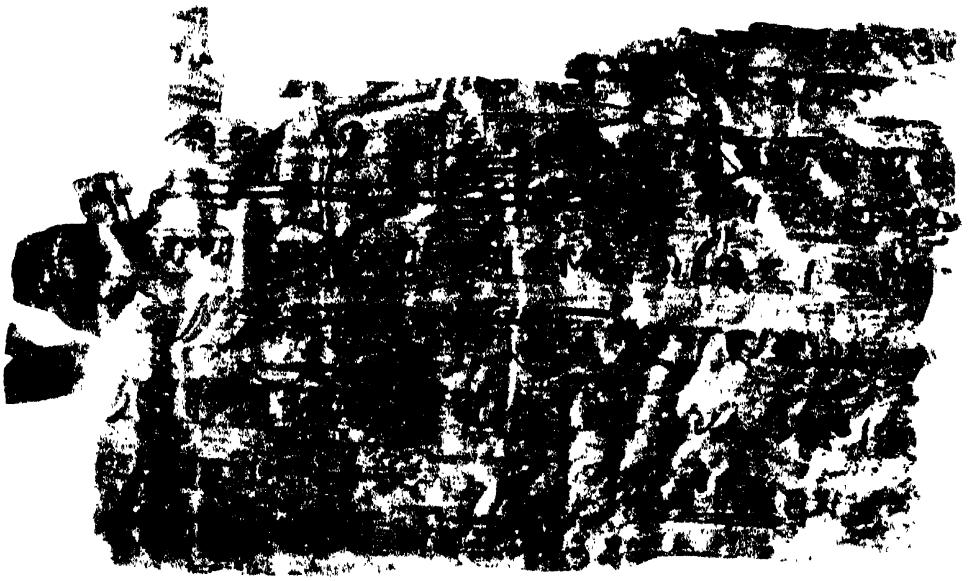


22 VERSO

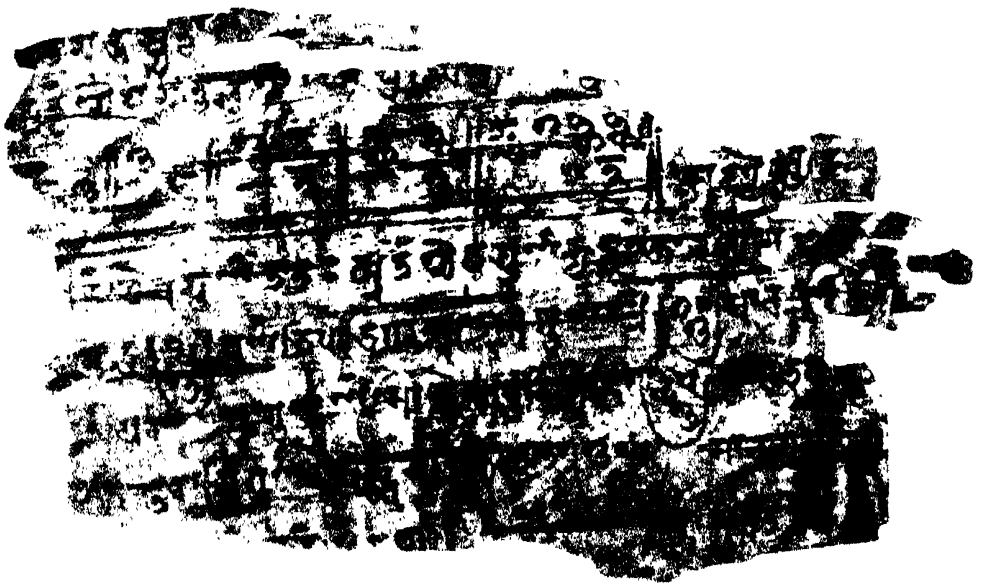
Plate XVI

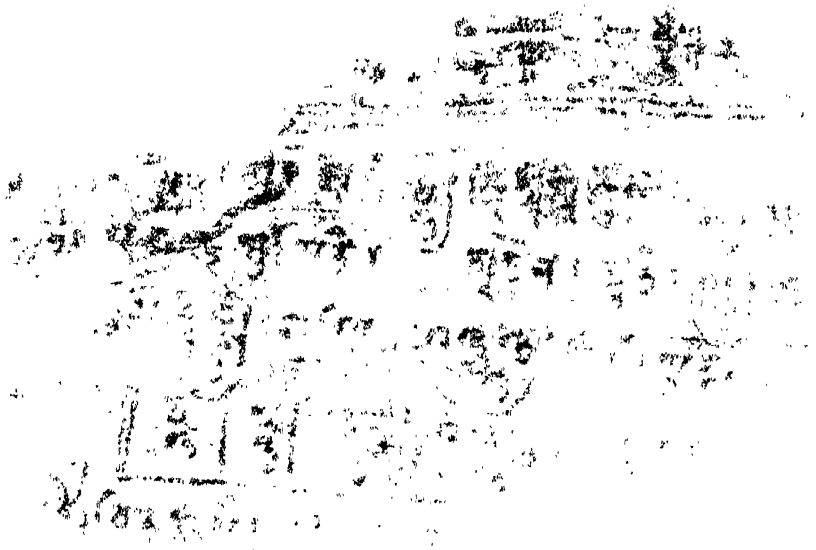
25 RECTO

25 VERSO



24 RECTO





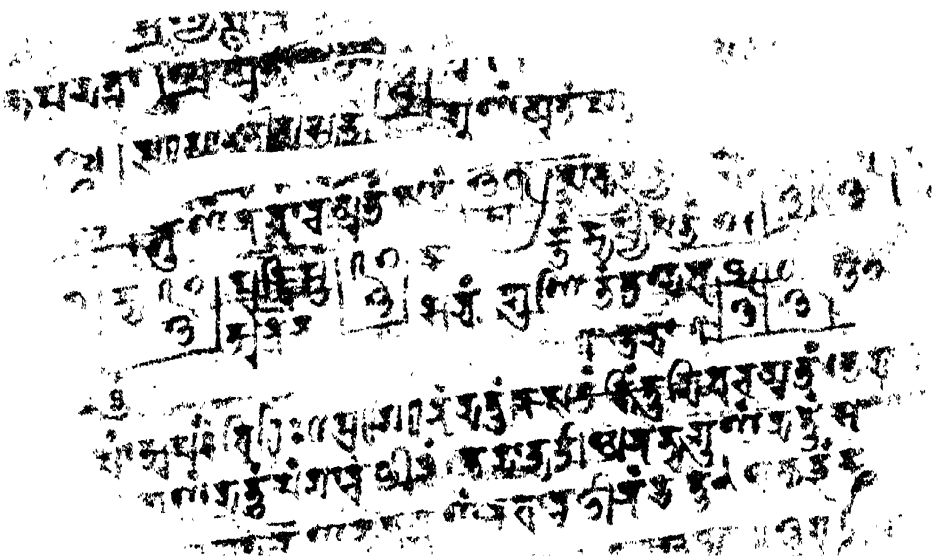
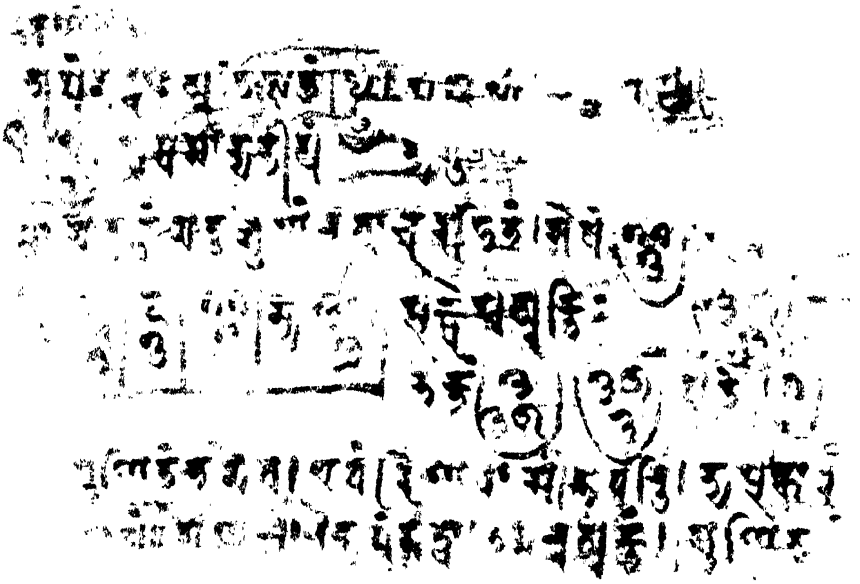
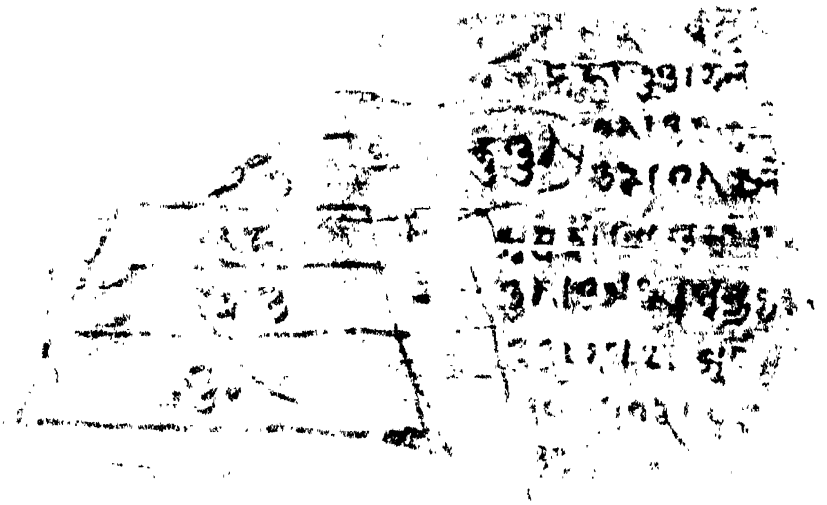


Plate XVIII

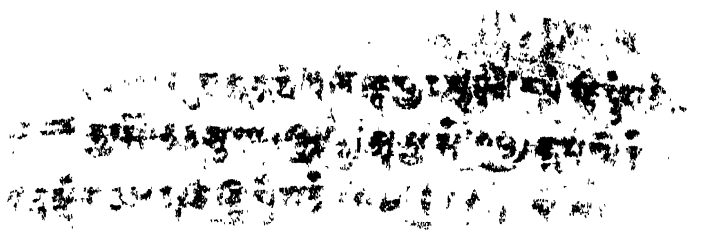
25 RECTO

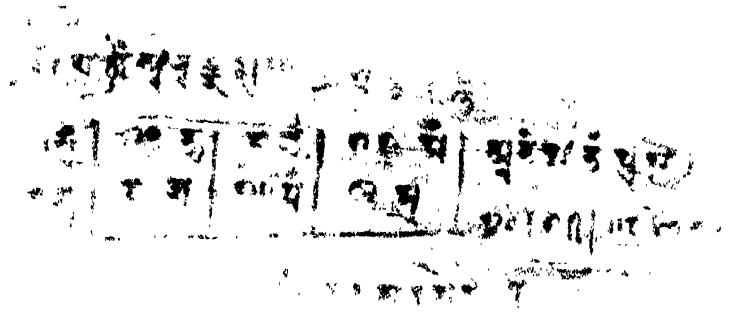


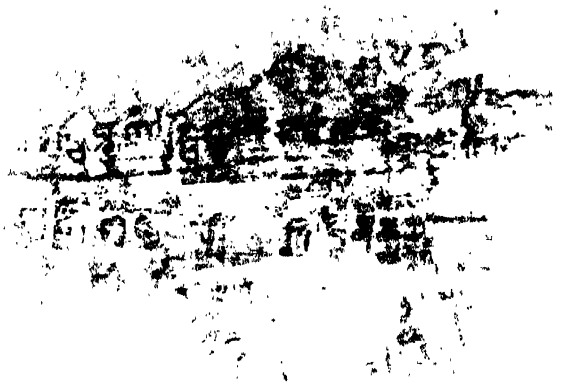
26 VERSO



27 RECTO







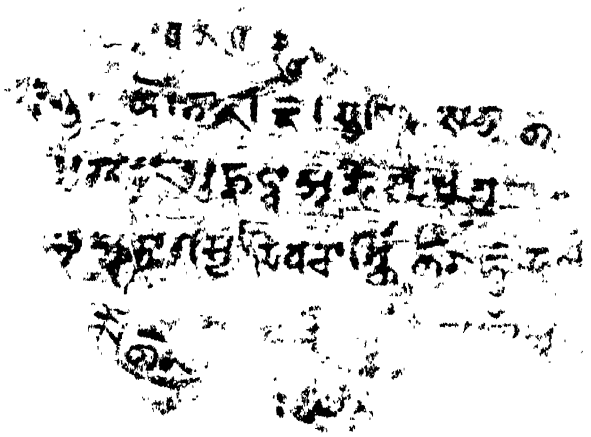
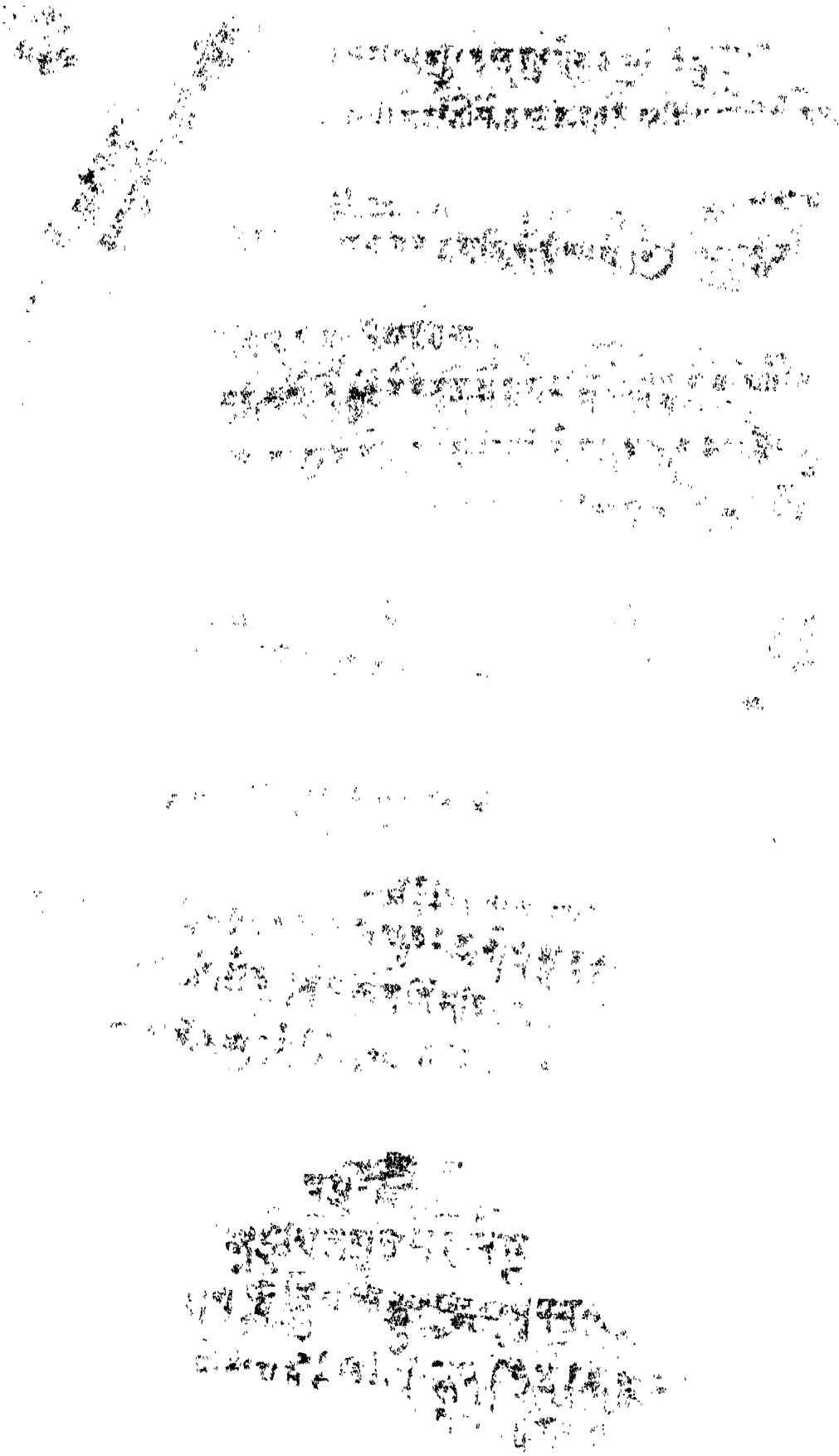
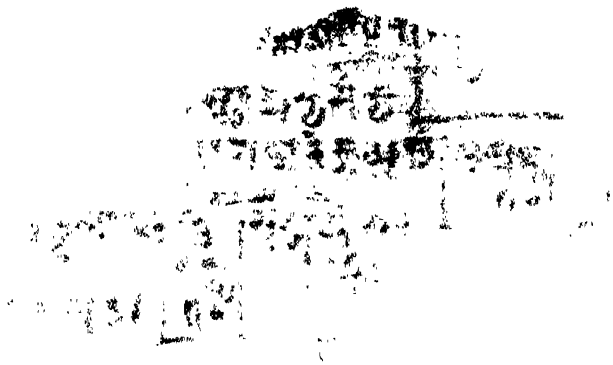


Plate XX





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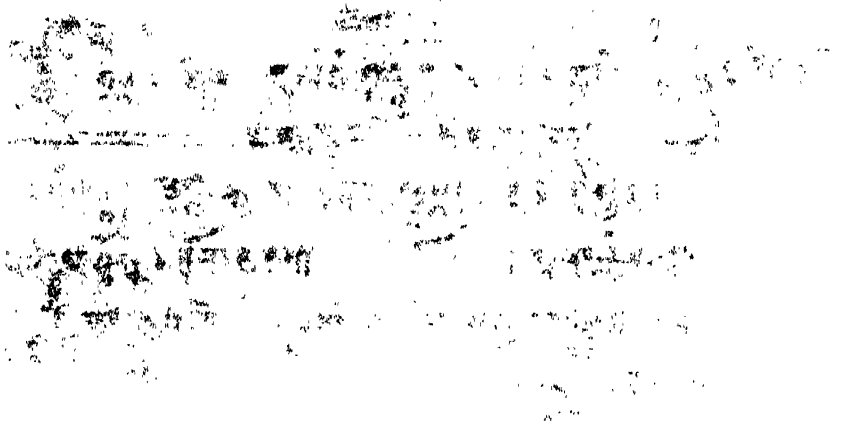

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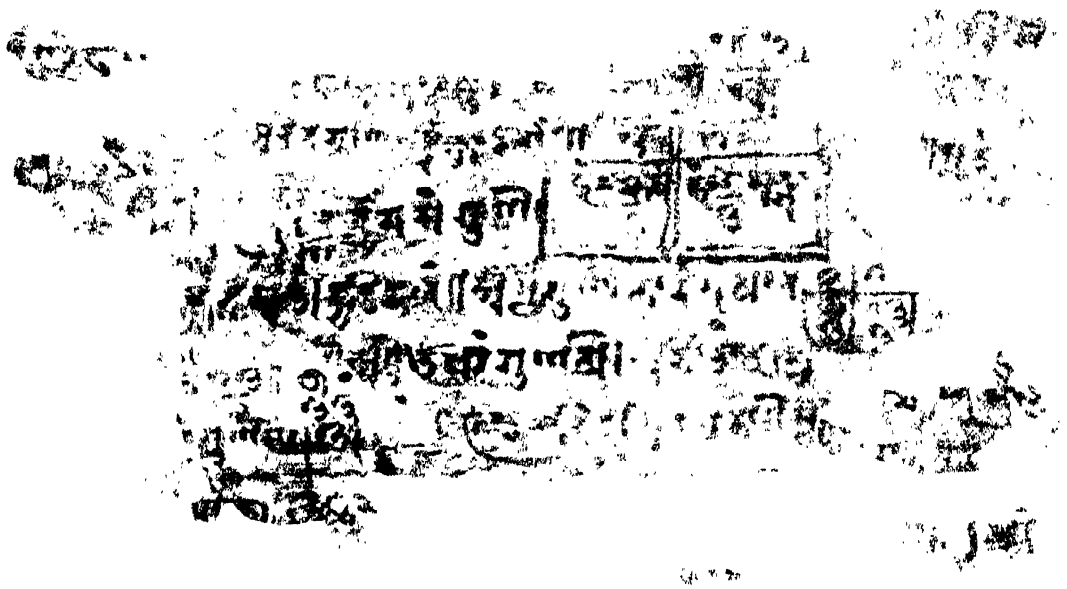

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Plate XXII

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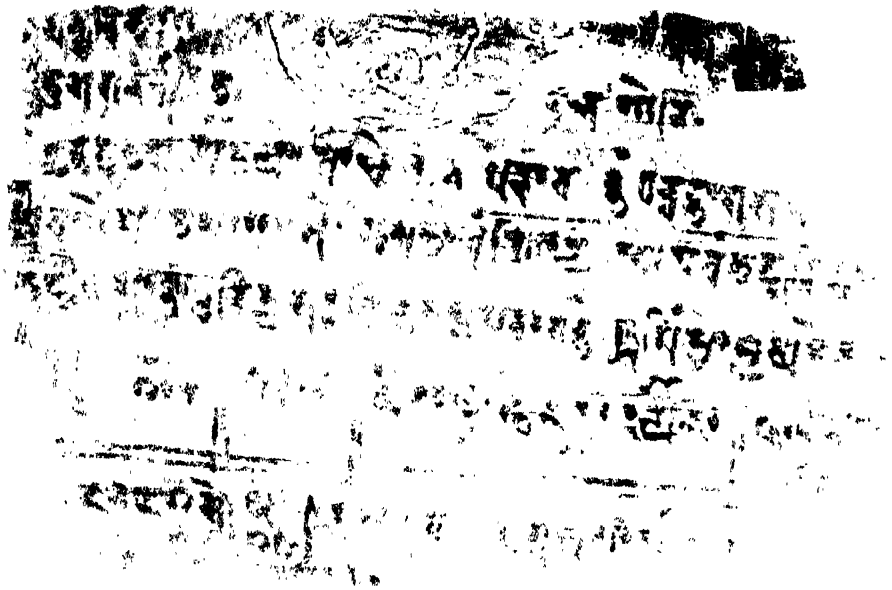

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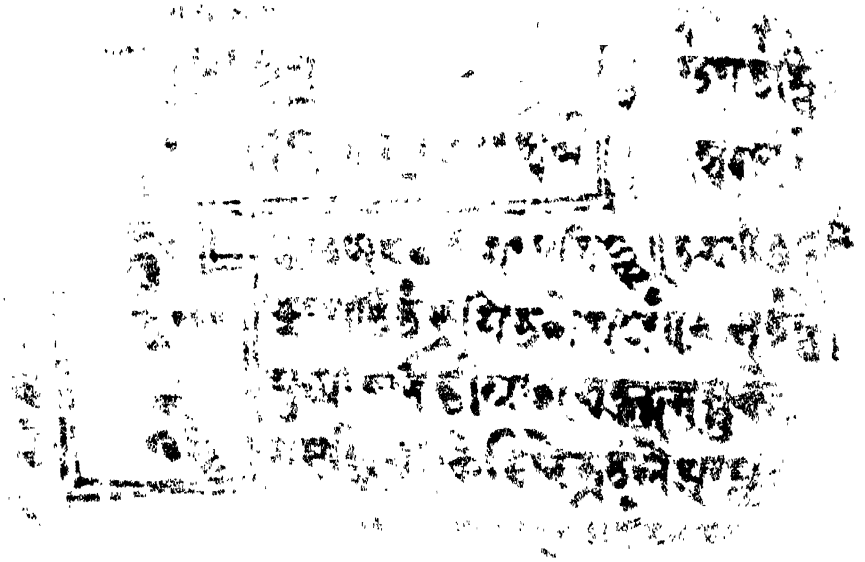

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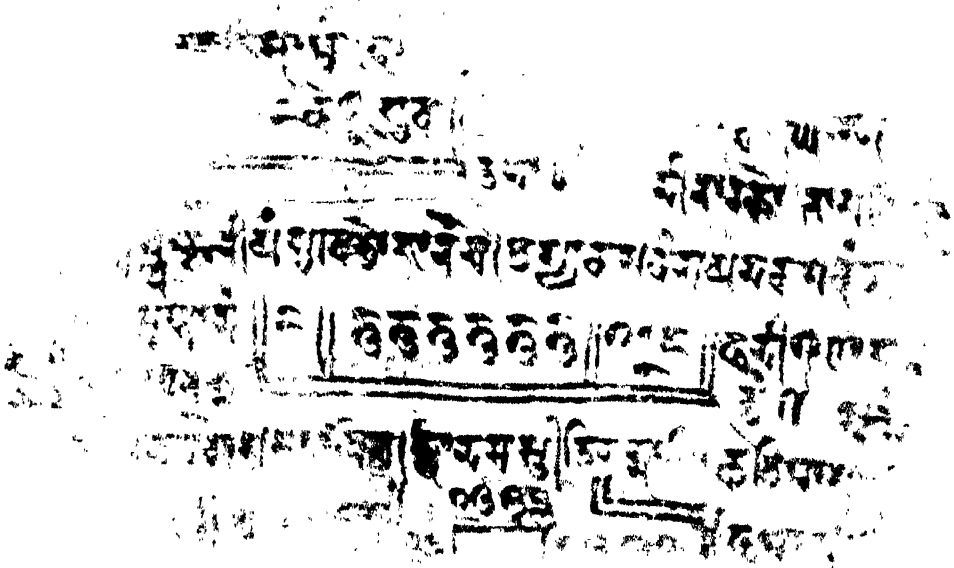

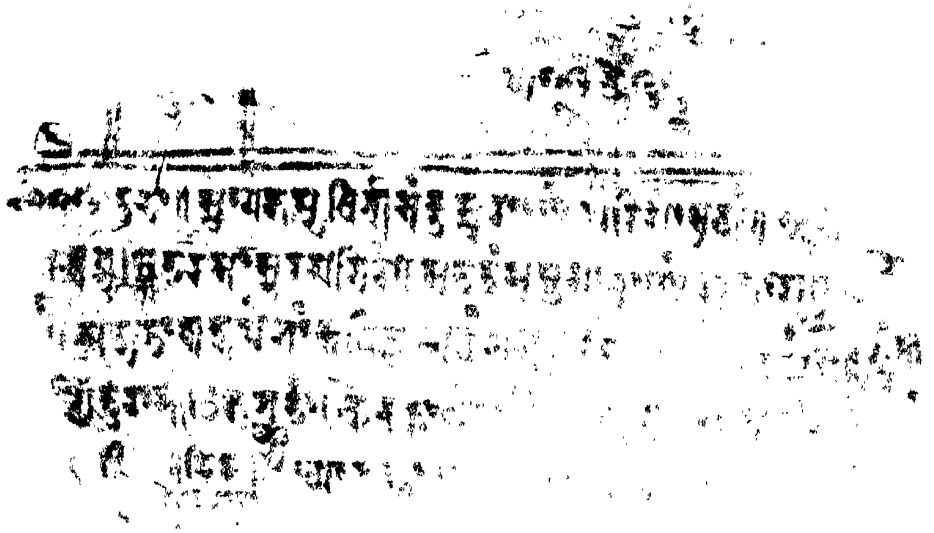

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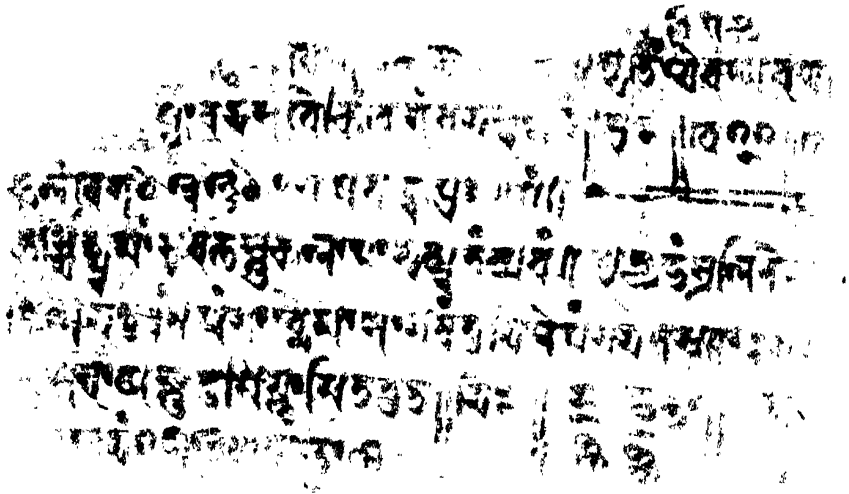

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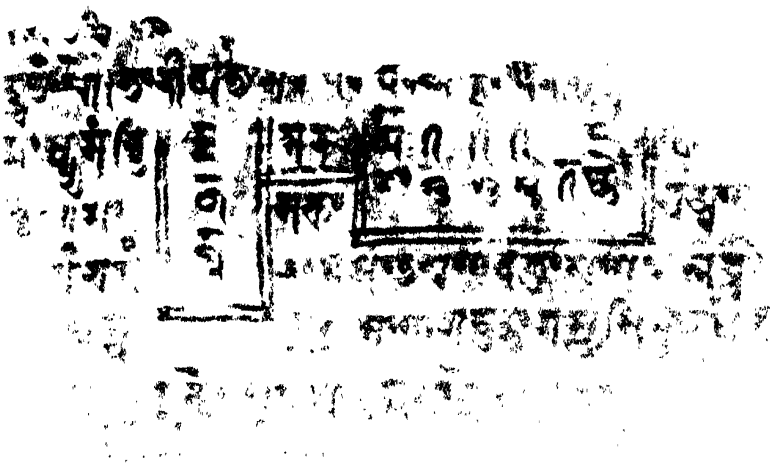

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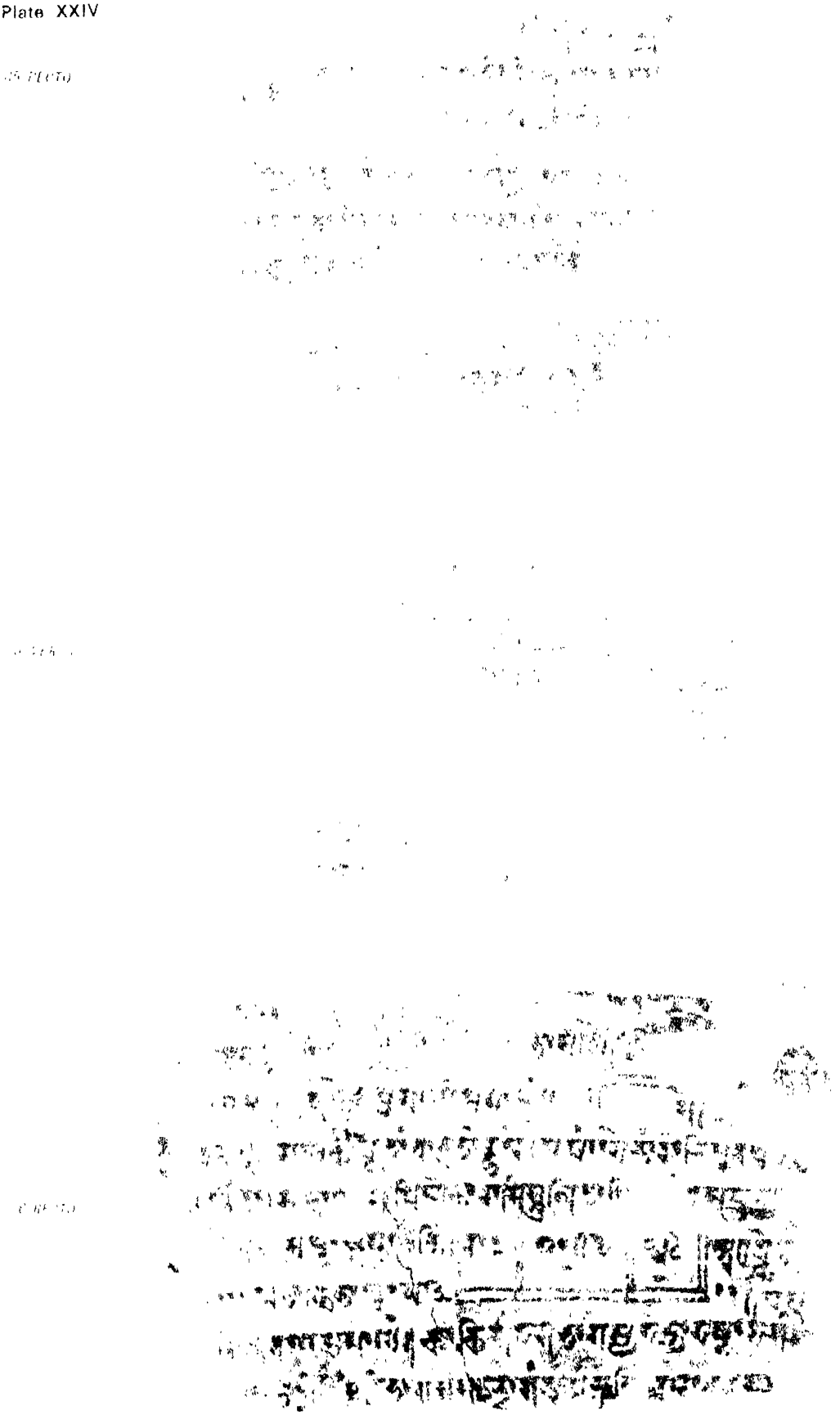

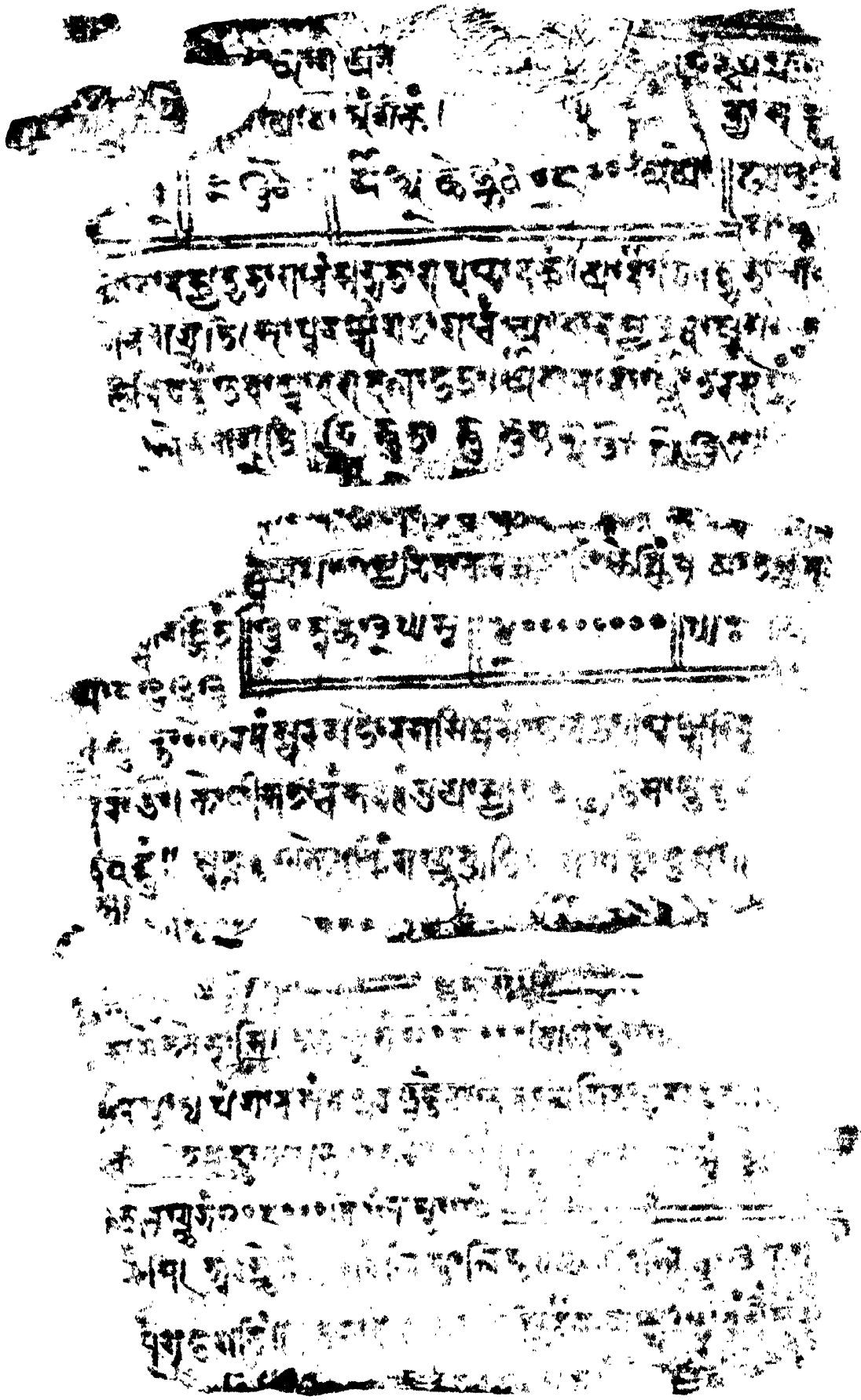

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Plate XXVI

3 RECTO

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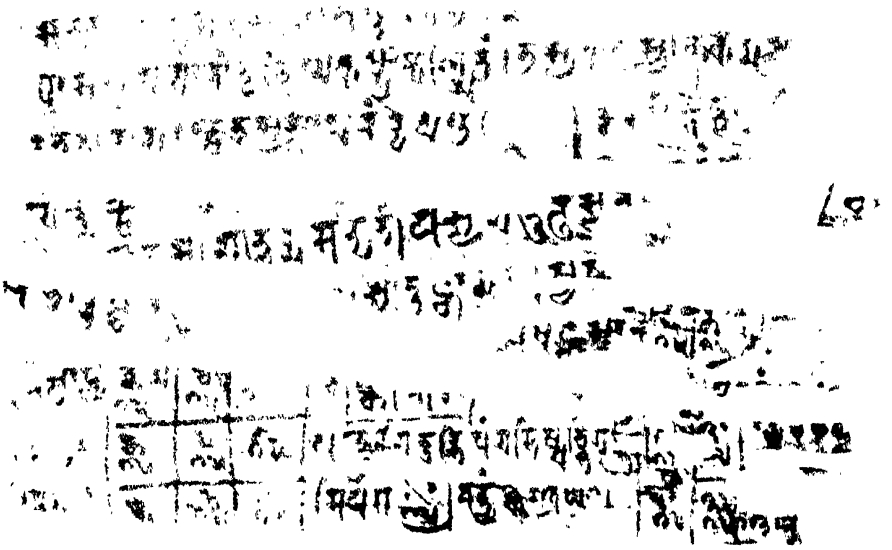

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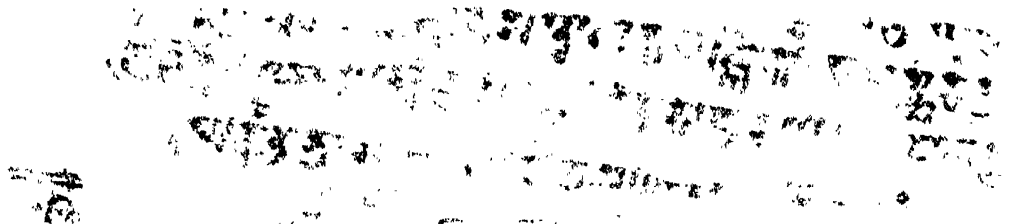

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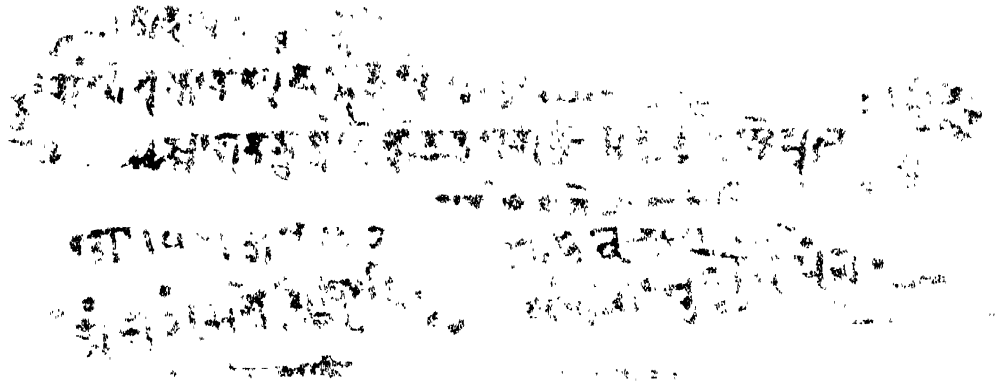

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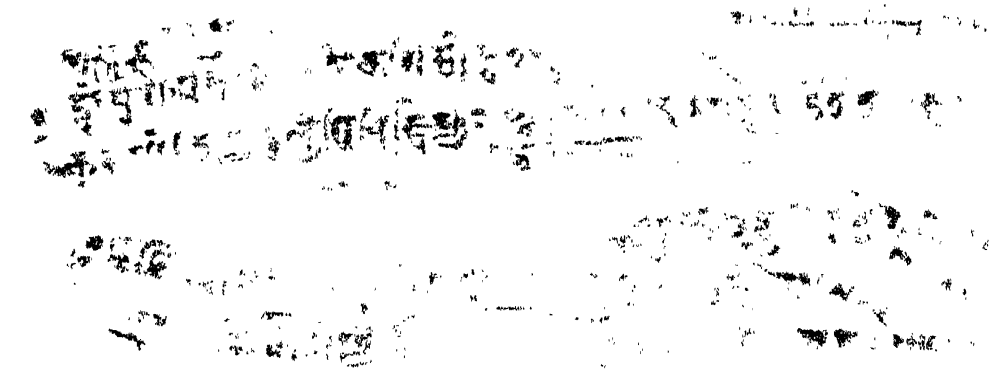

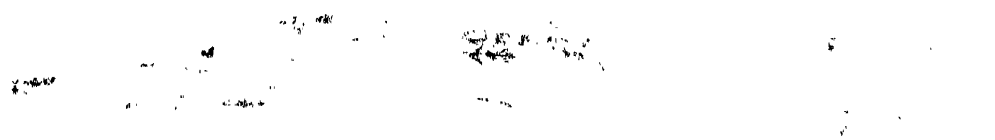

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Plate XXVIII

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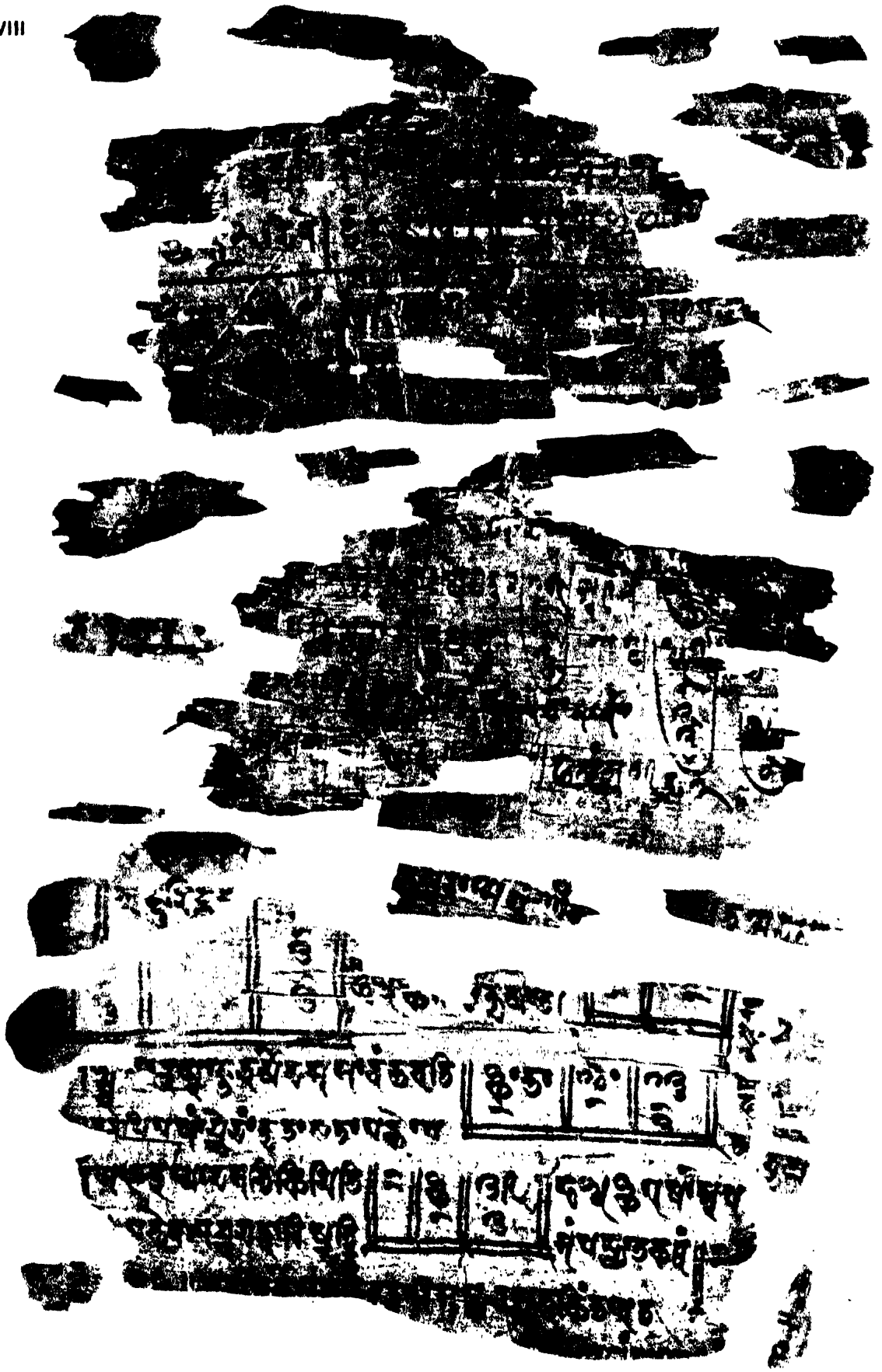

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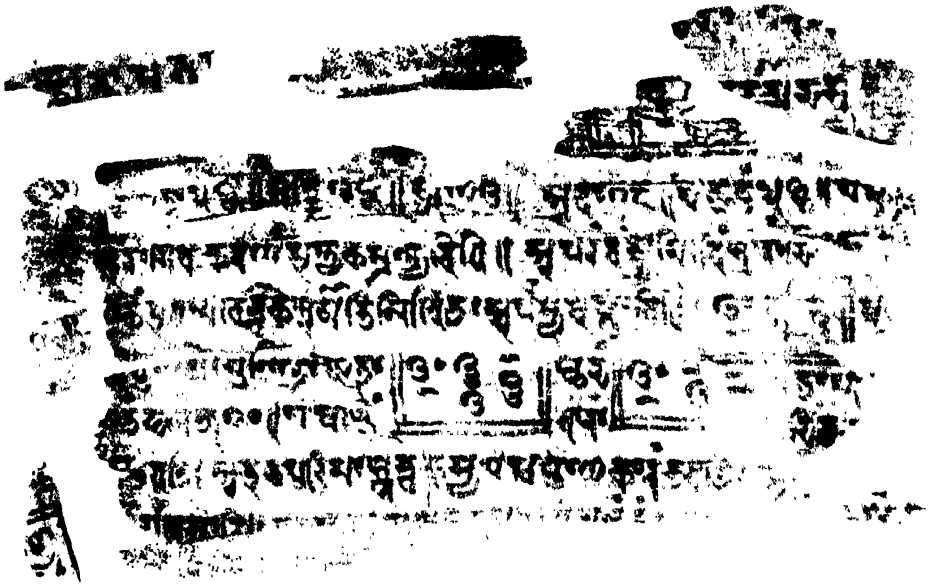

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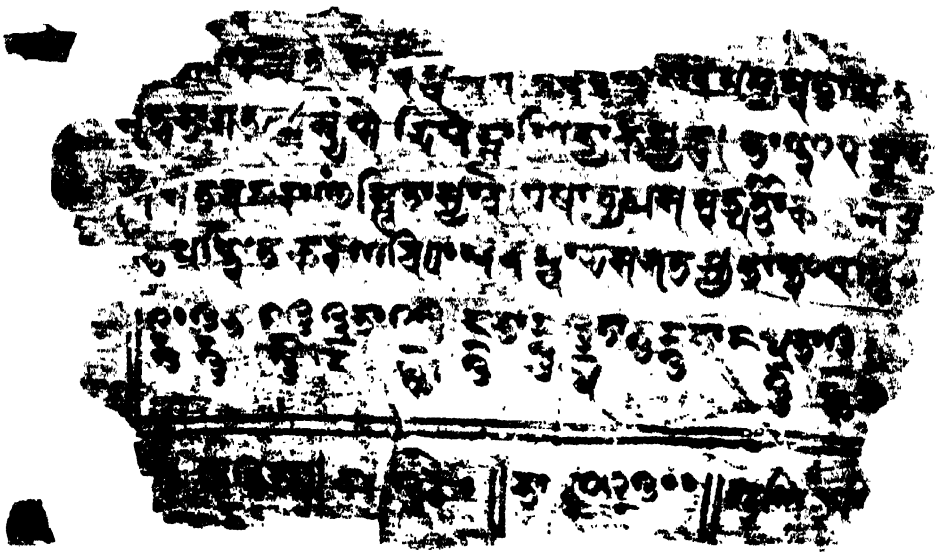

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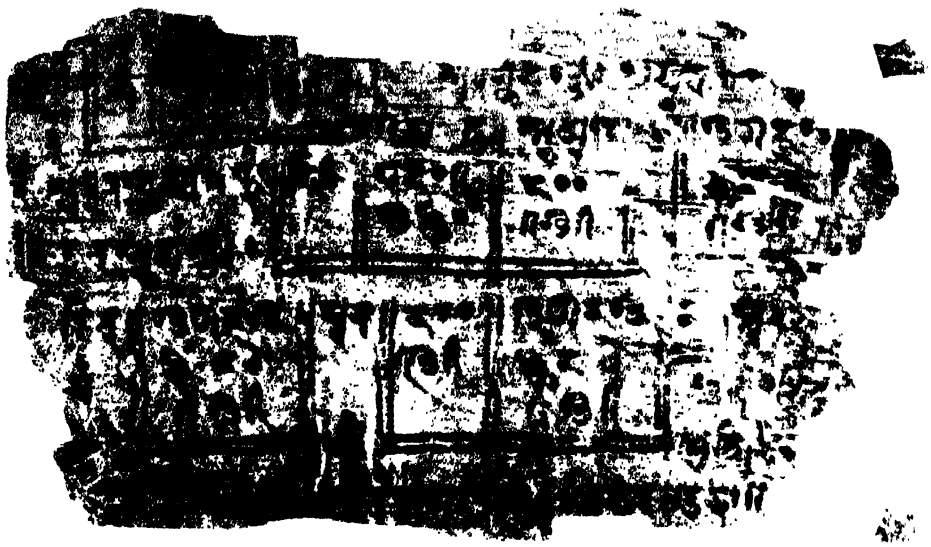

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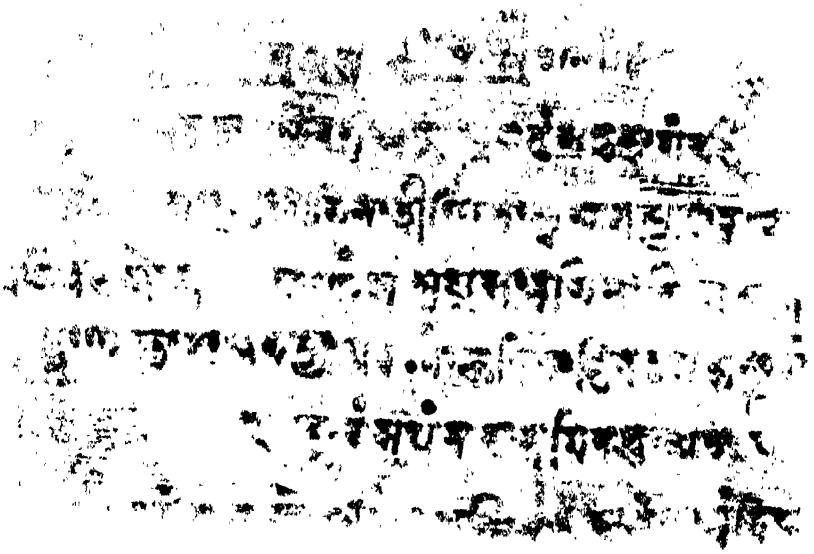
Plate XXX

44 RECTO



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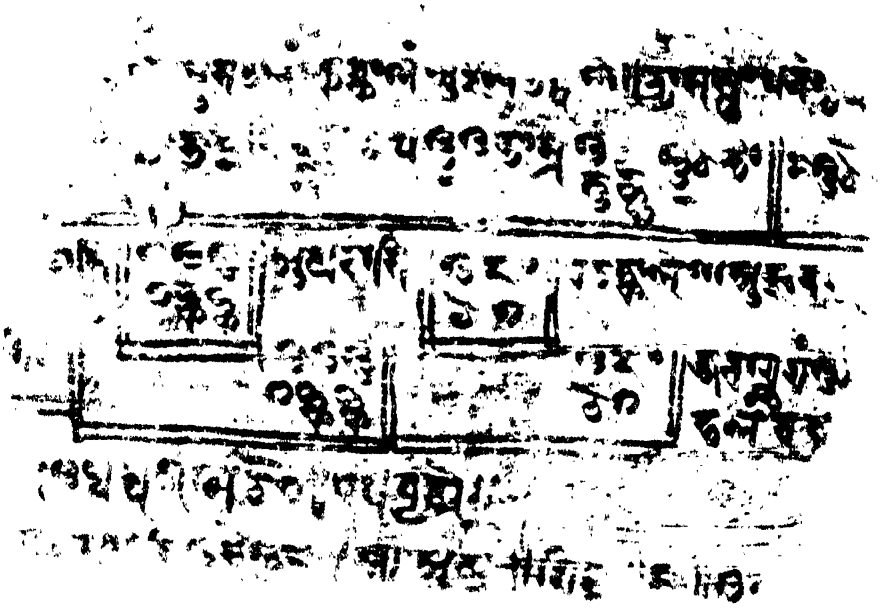

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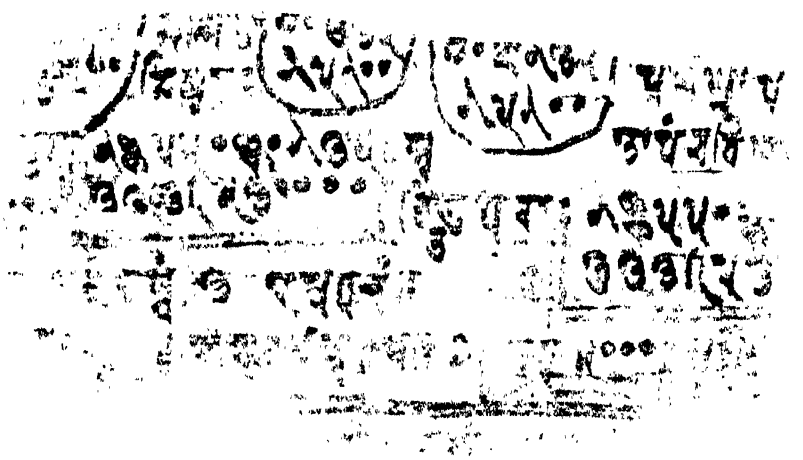

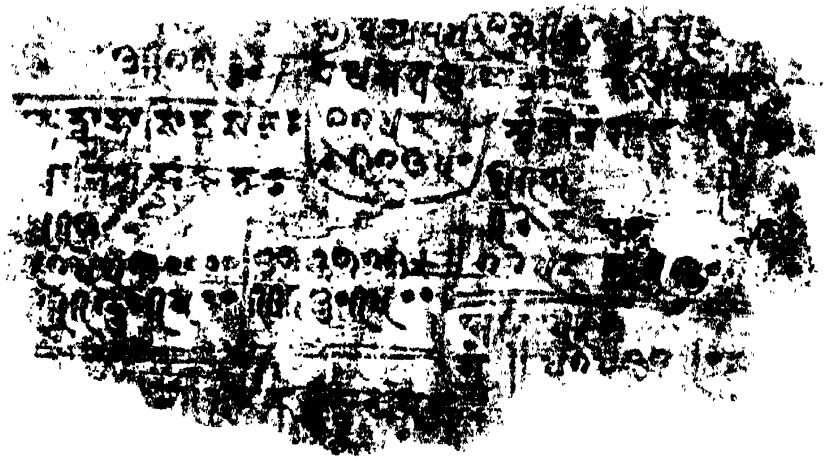

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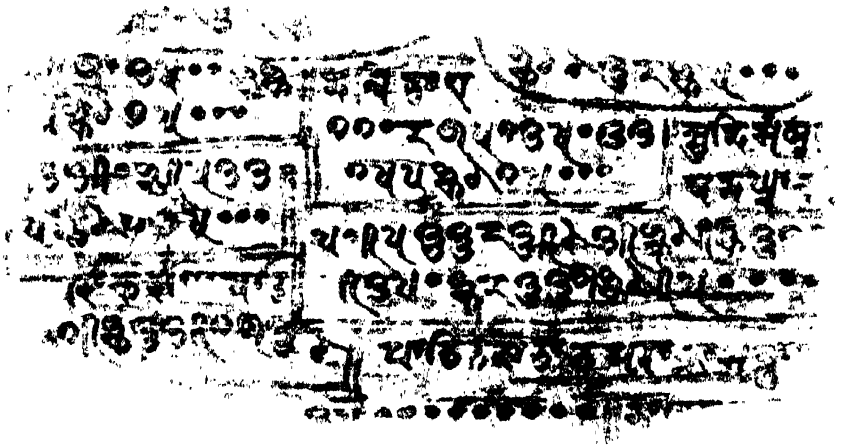

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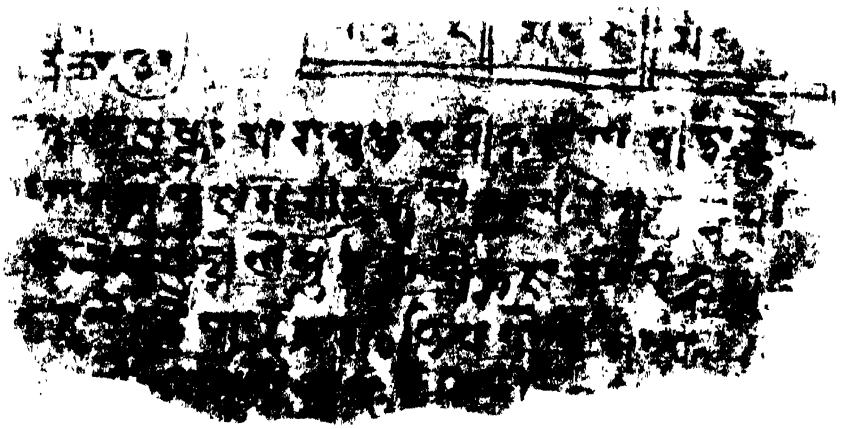

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Plate XXXII

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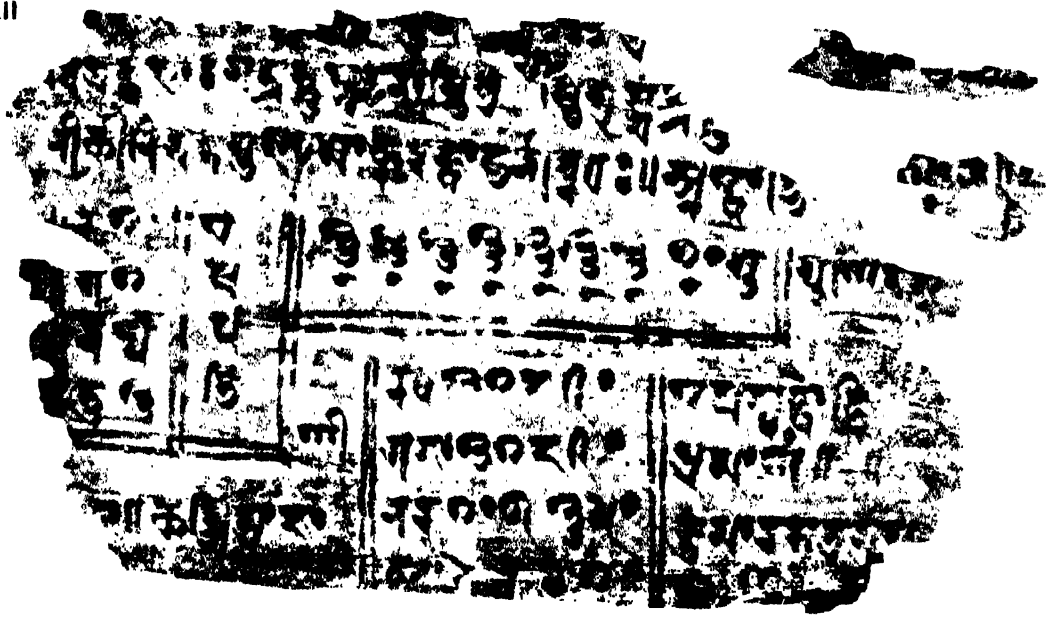

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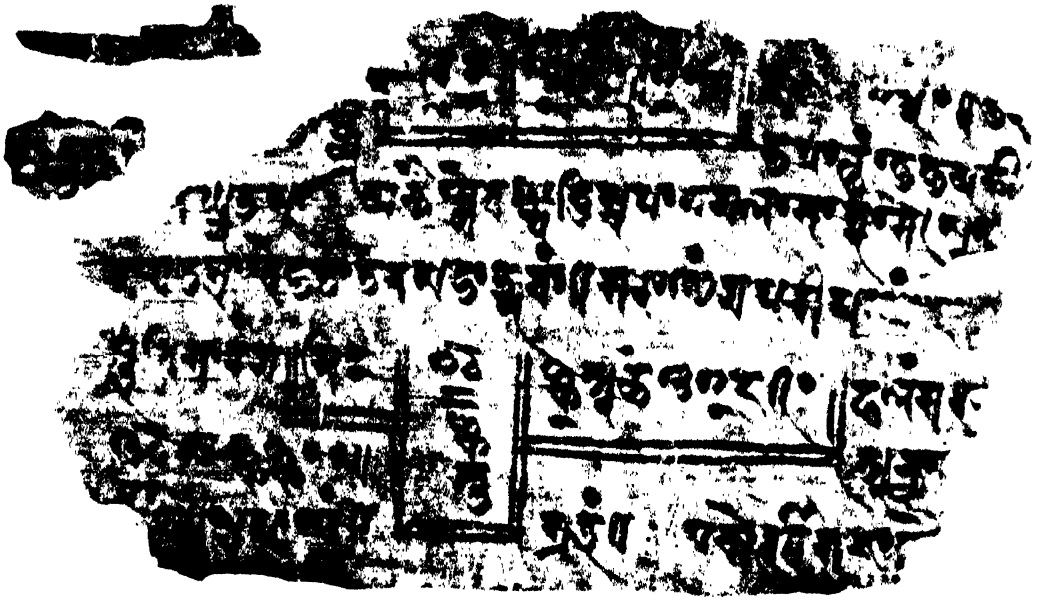

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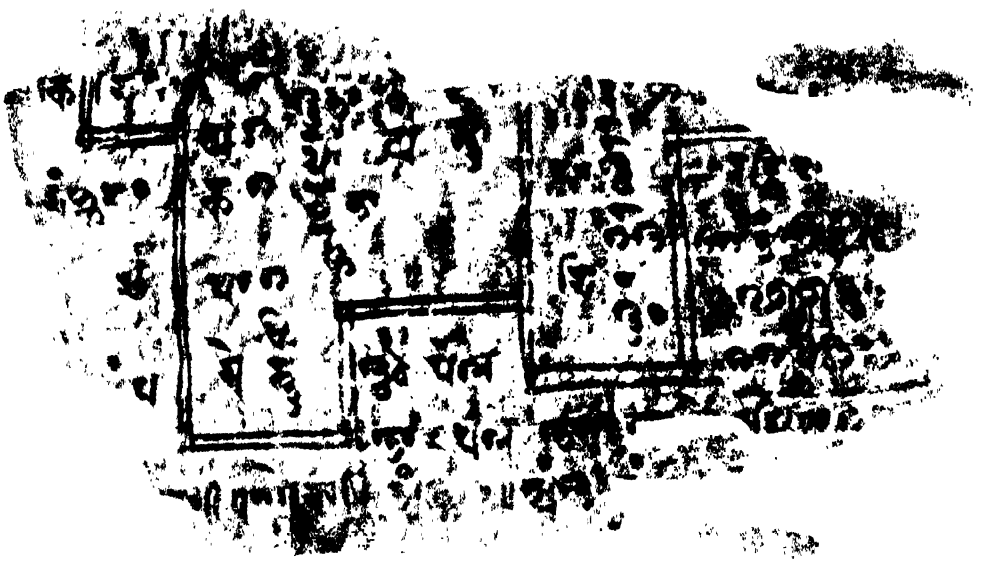

Plate XXXIII

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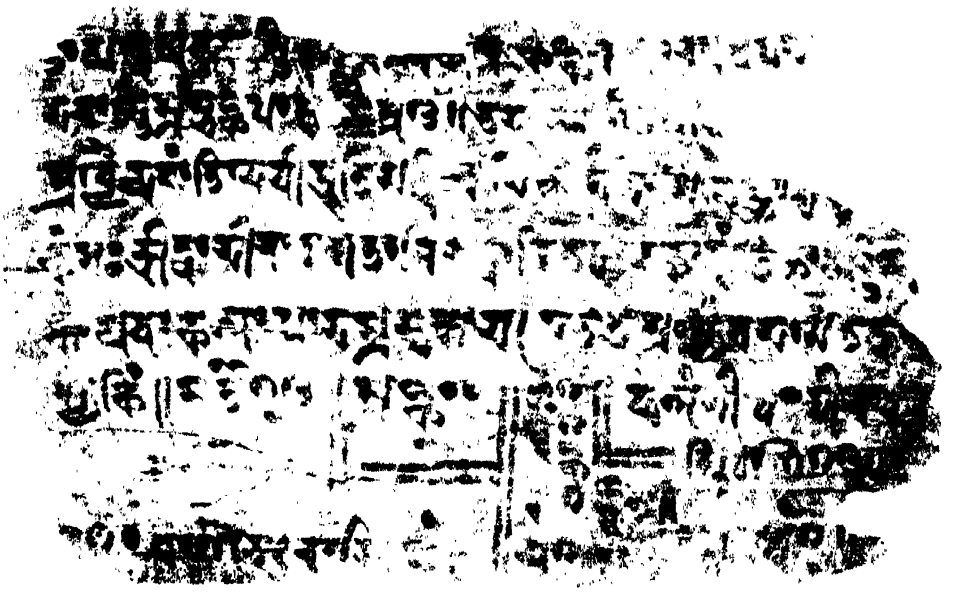

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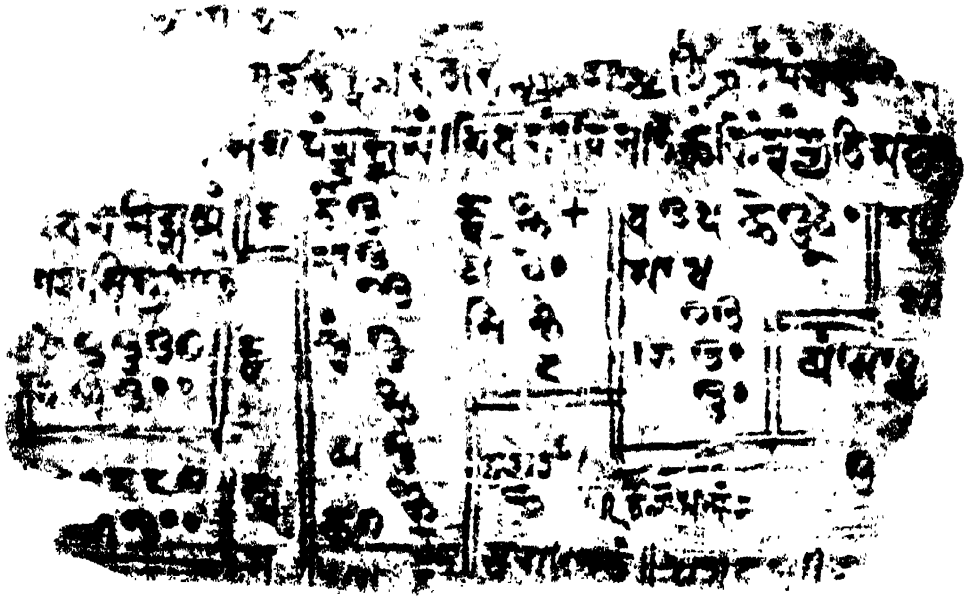

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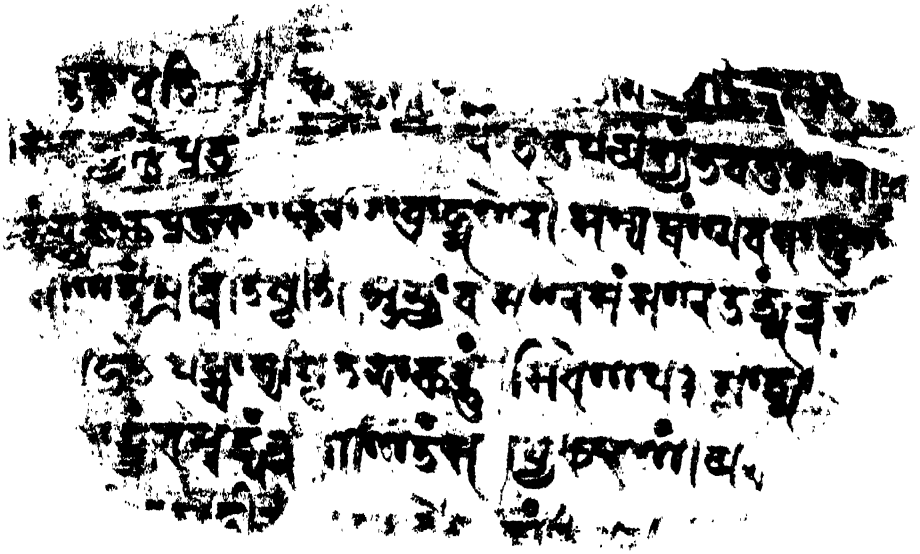

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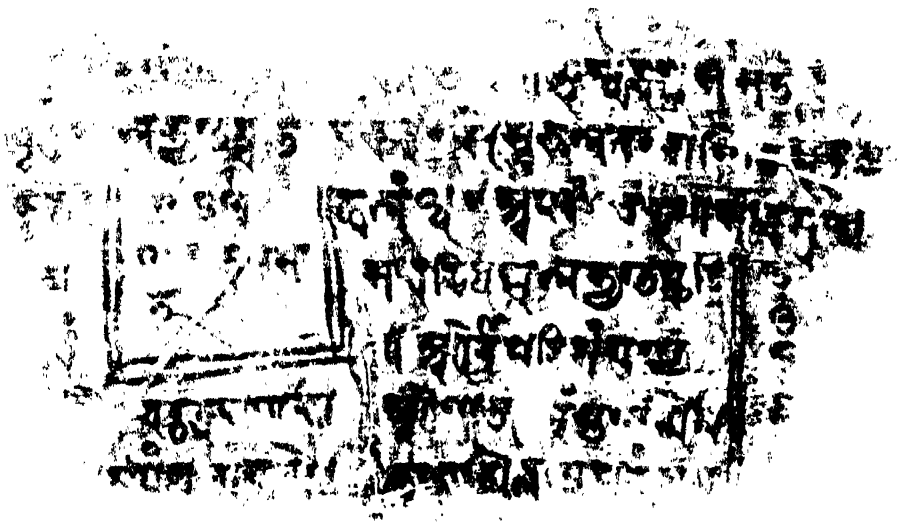

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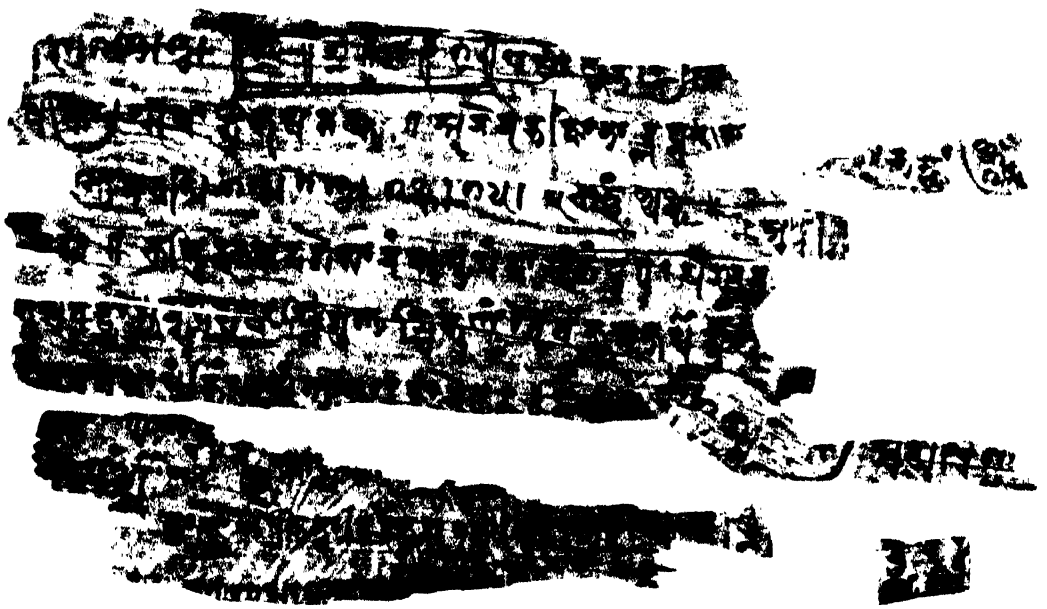

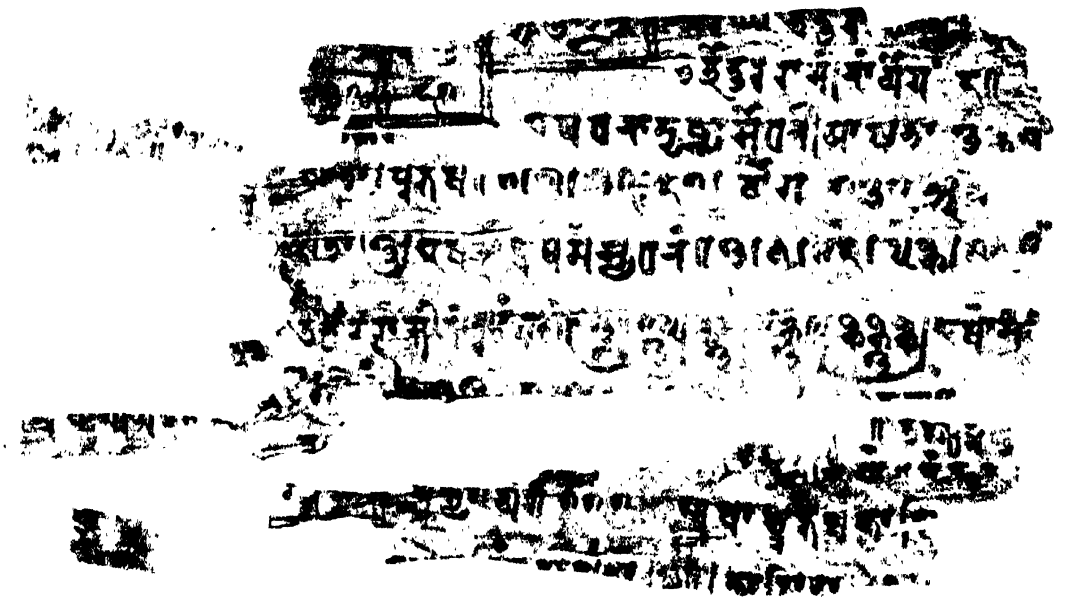

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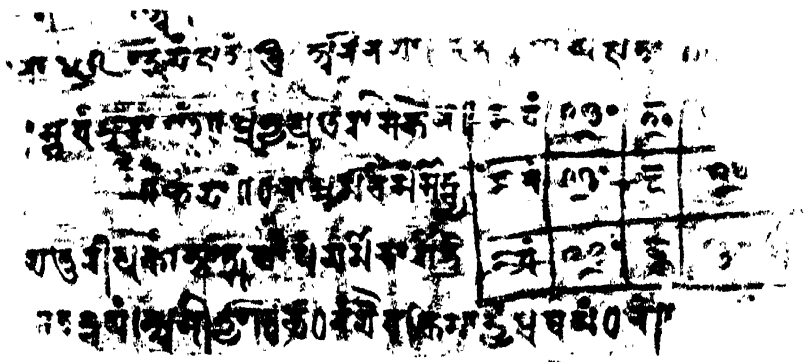

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Plate XXXVI

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Plate XXXVII

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Plate XXXVIII

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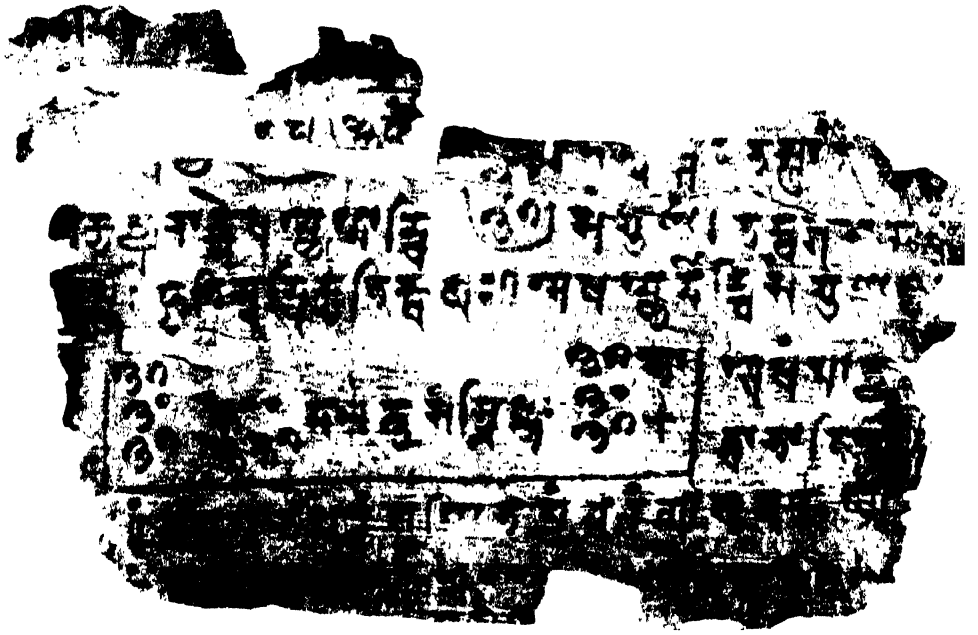

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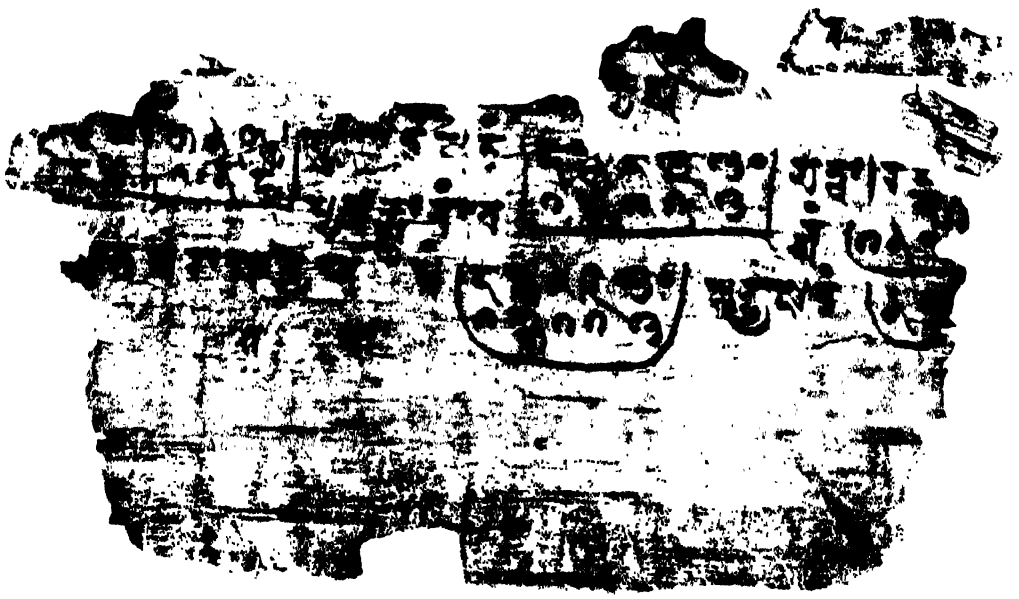

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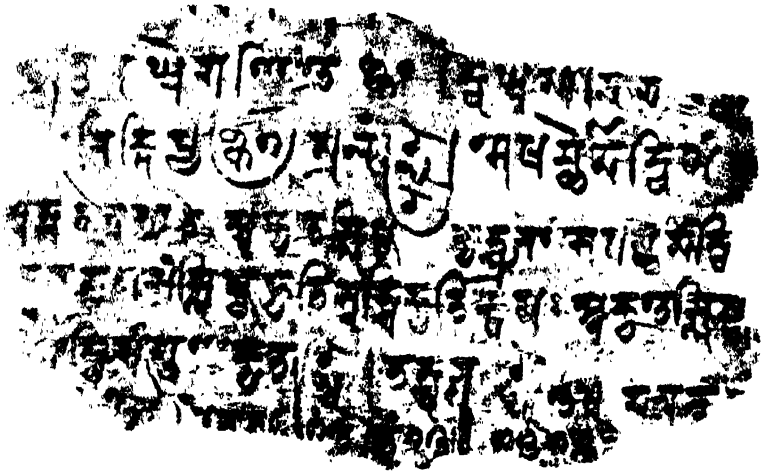

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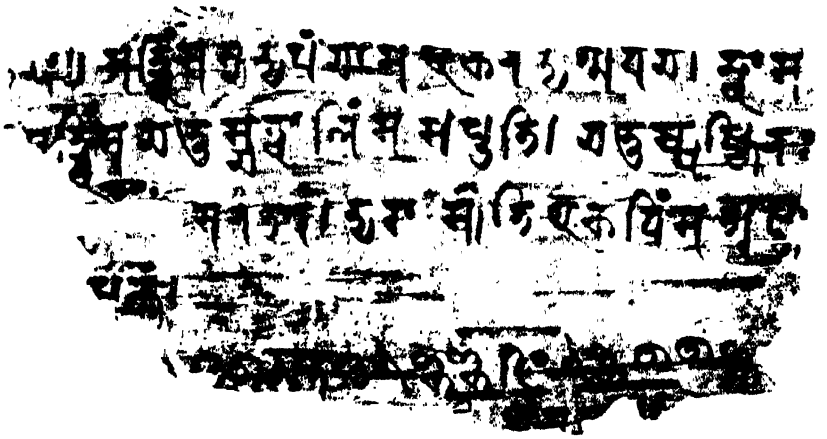

68 RECTO

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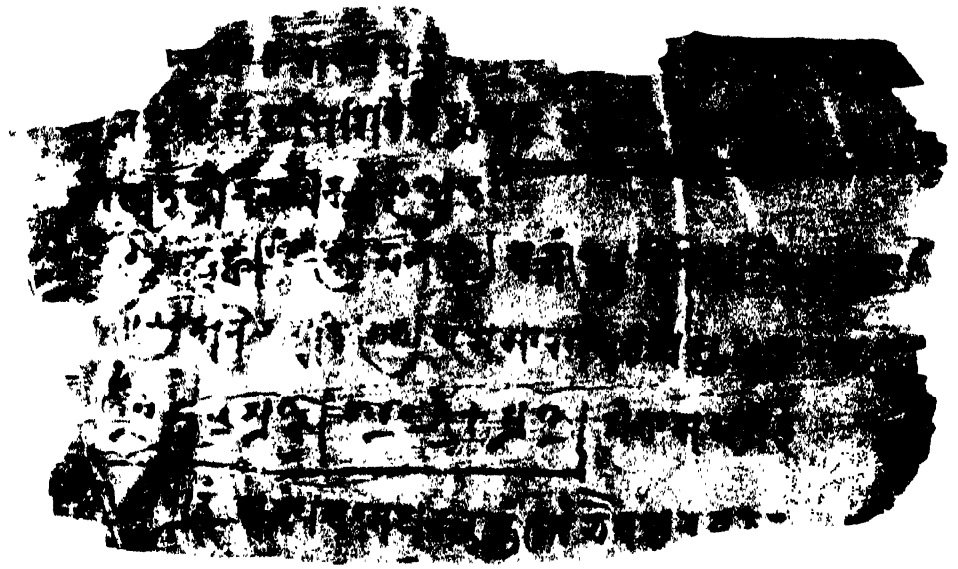
Plate XL

59 RECTO



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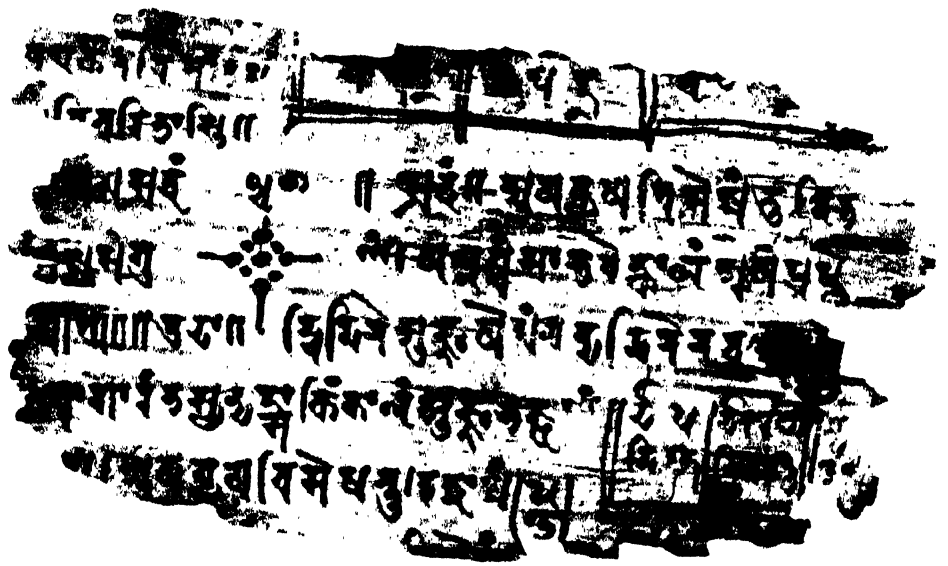

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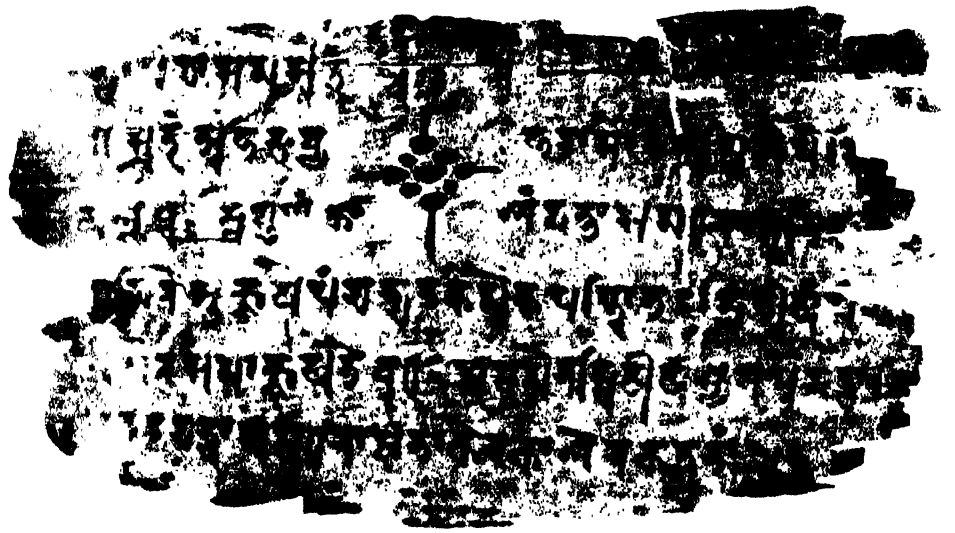

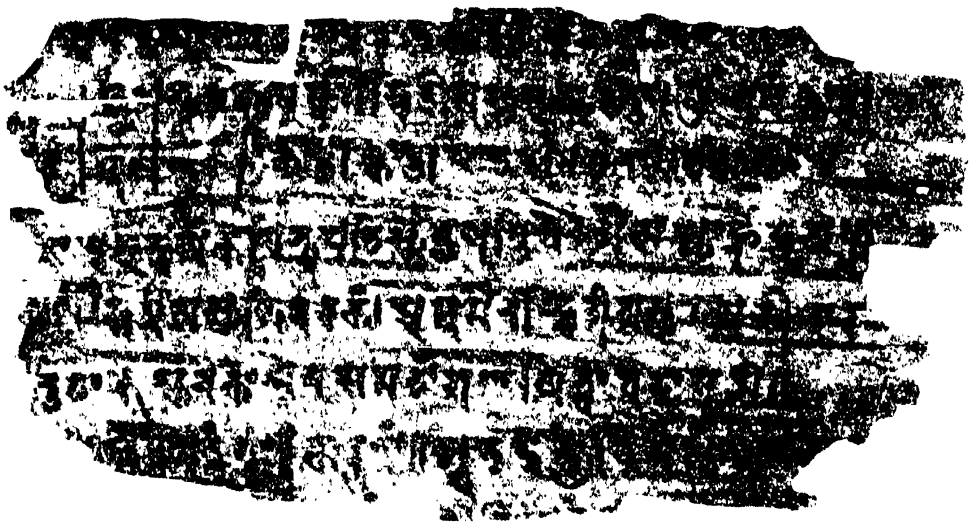

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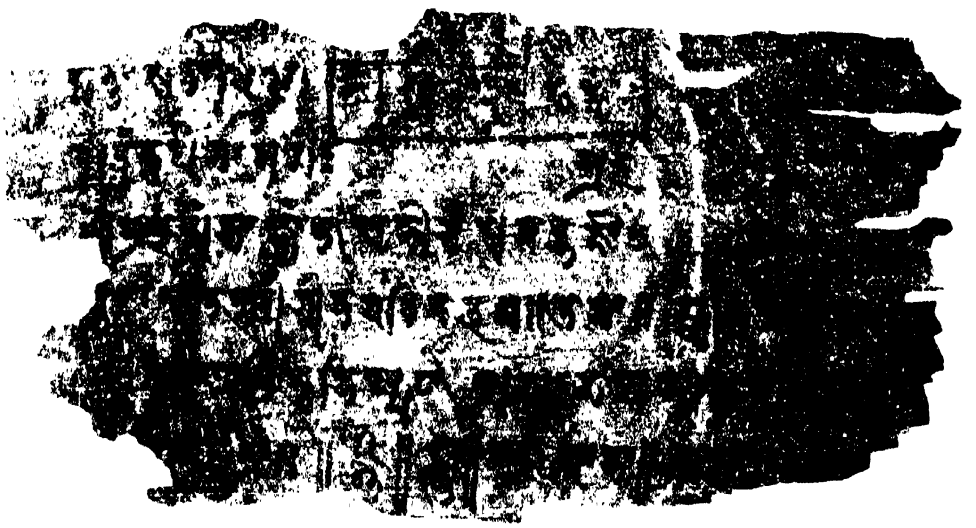

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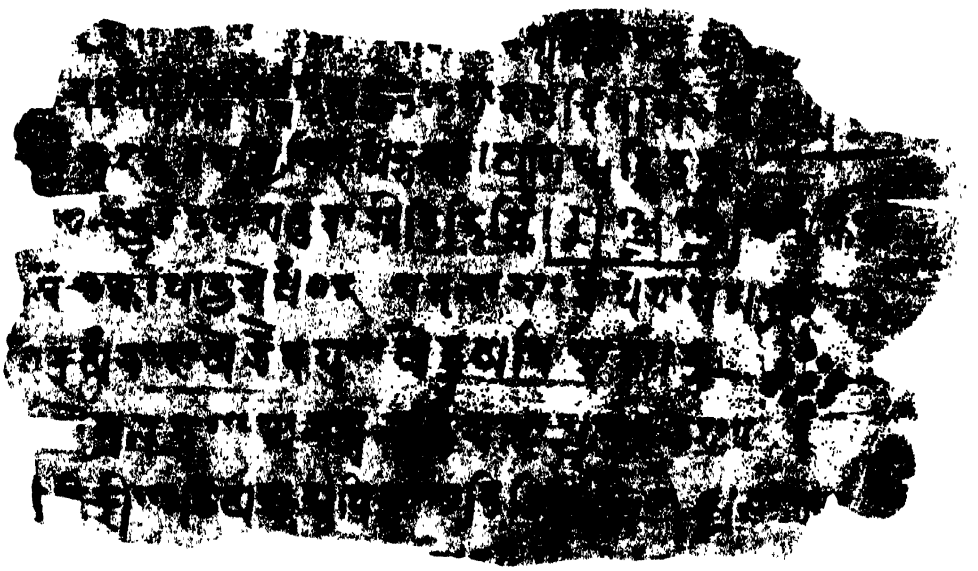

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Plate XLII

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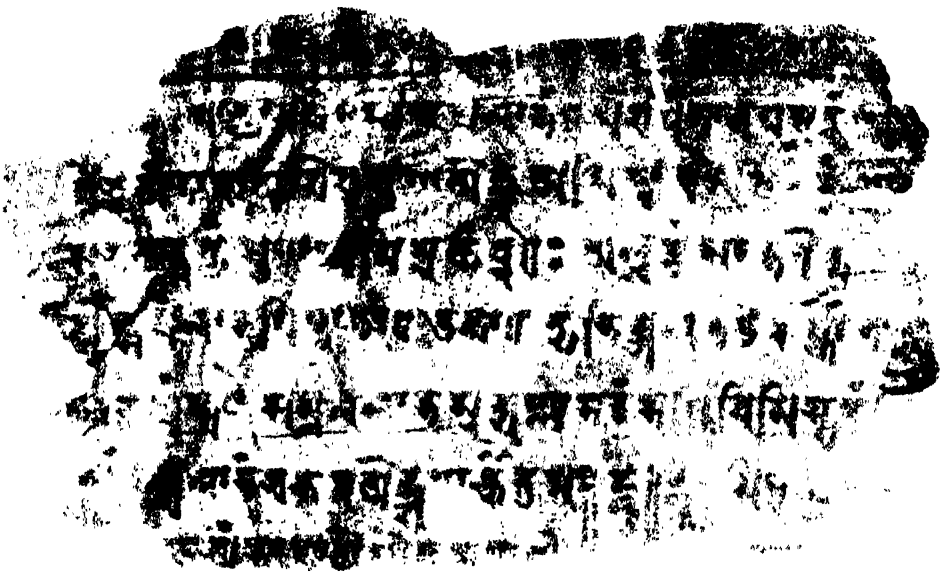

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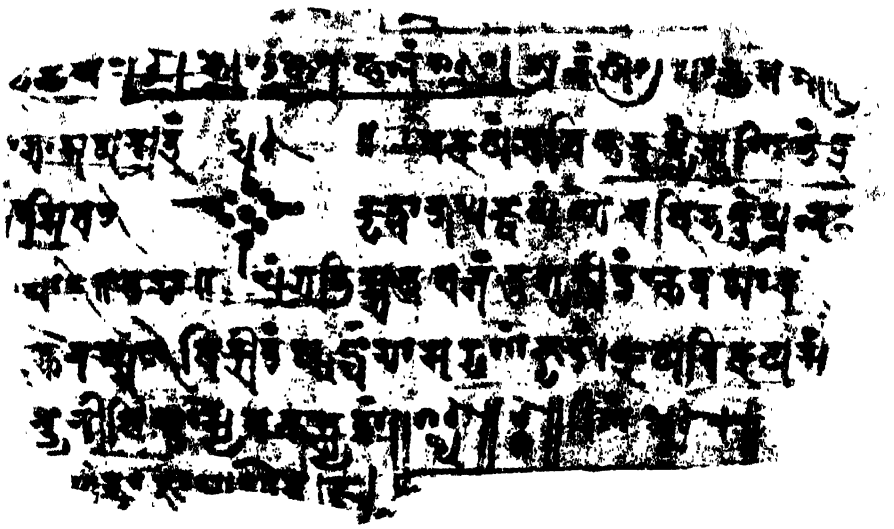

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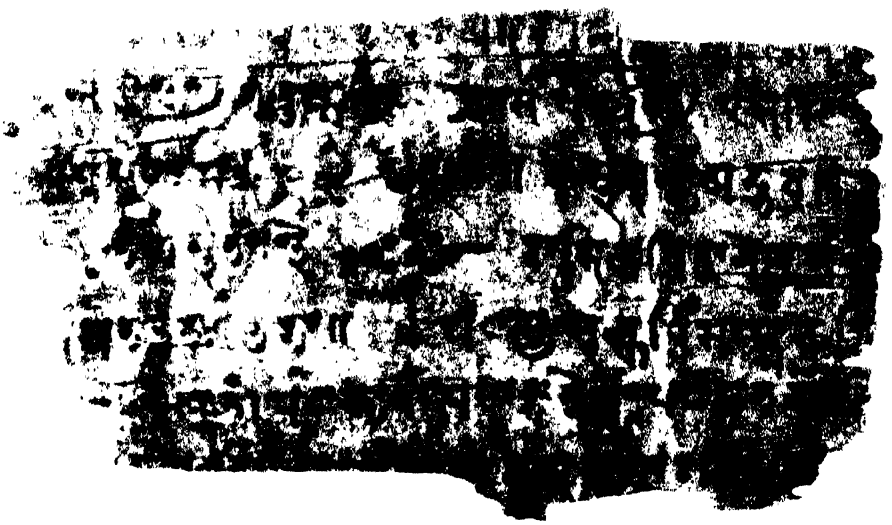

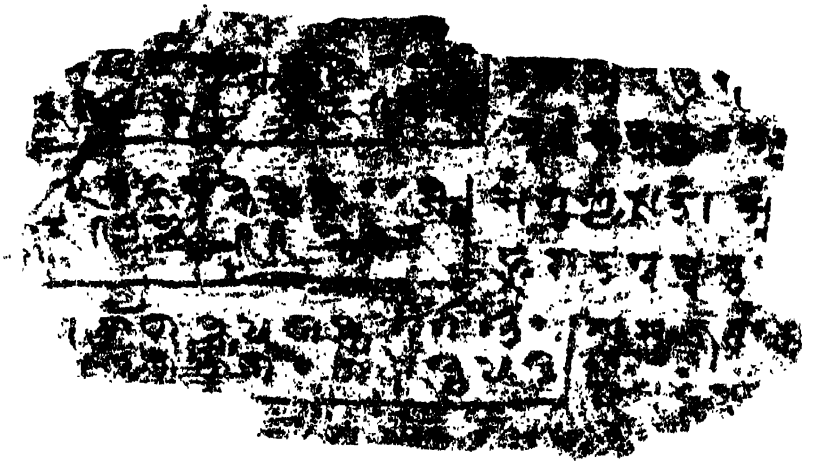

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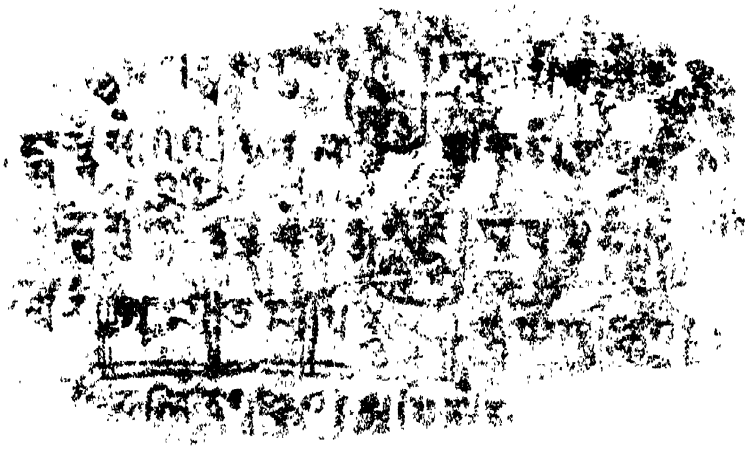

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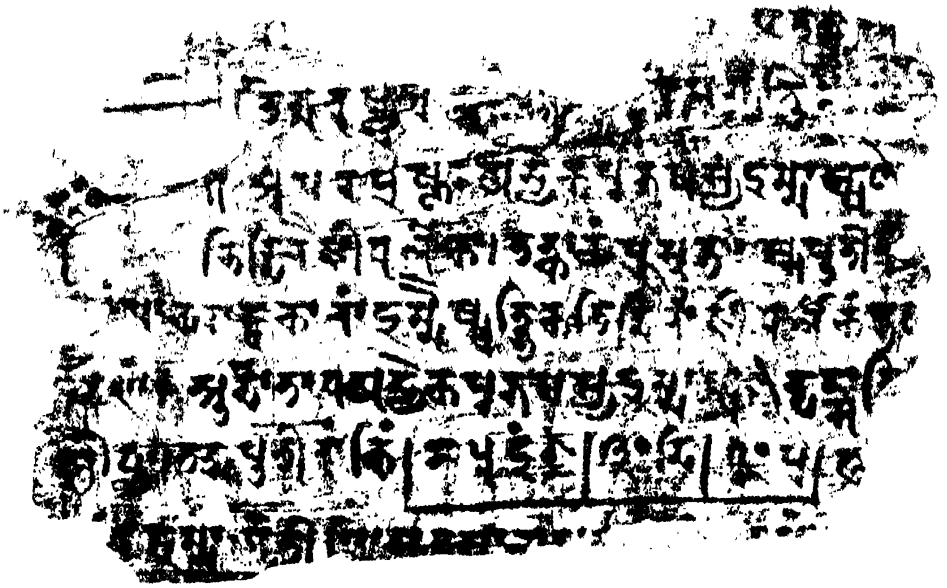

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Plate XLIV

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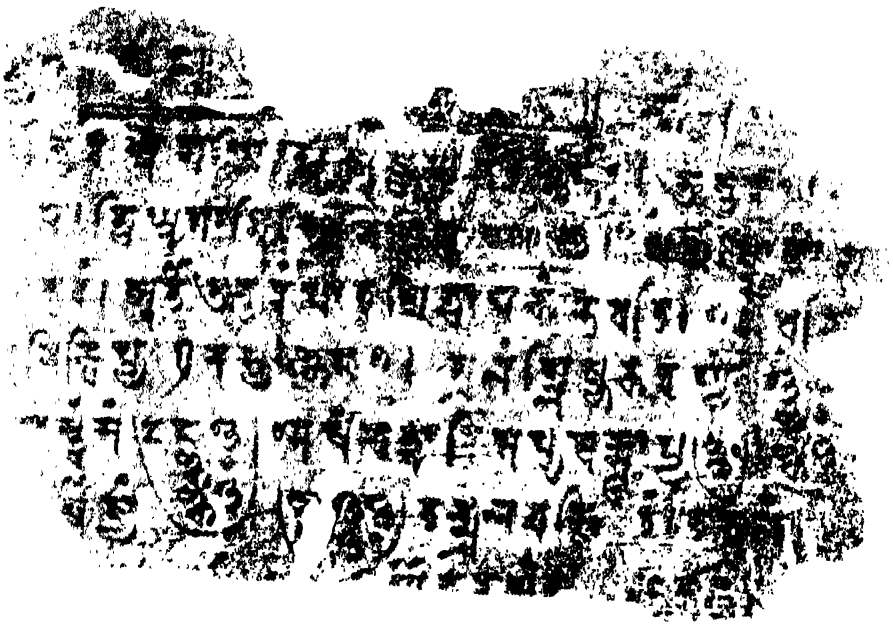

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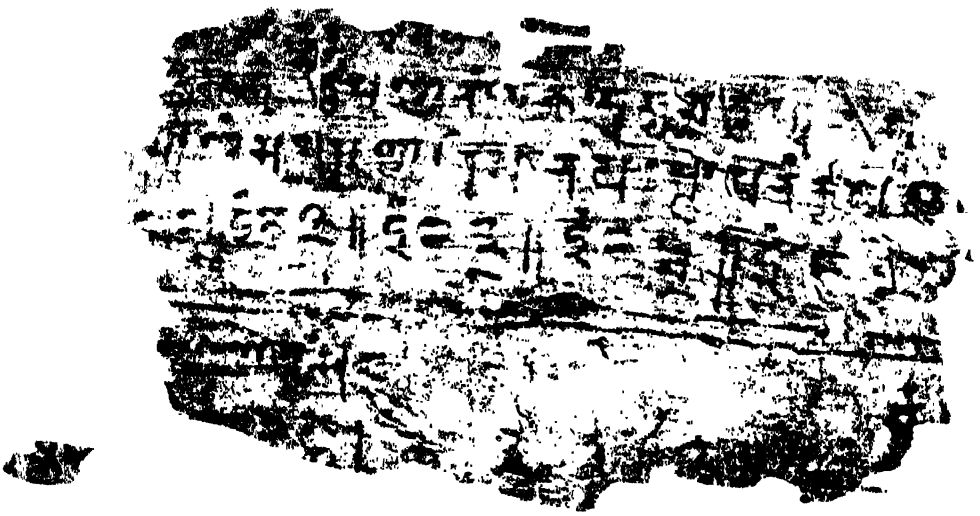

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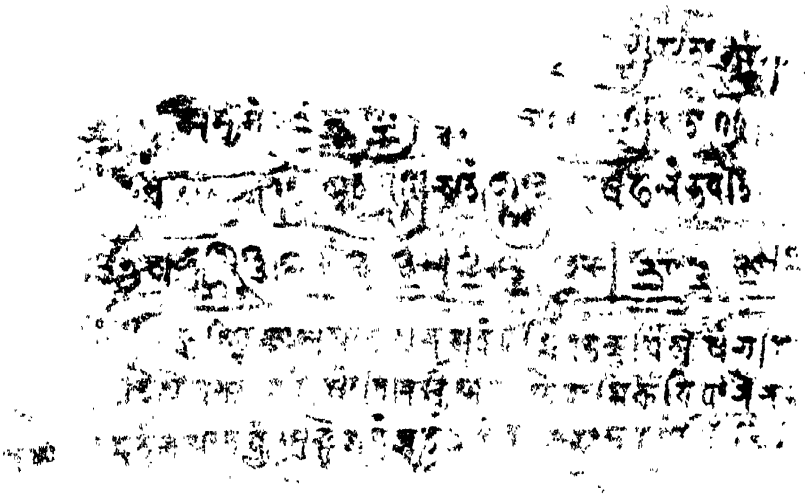

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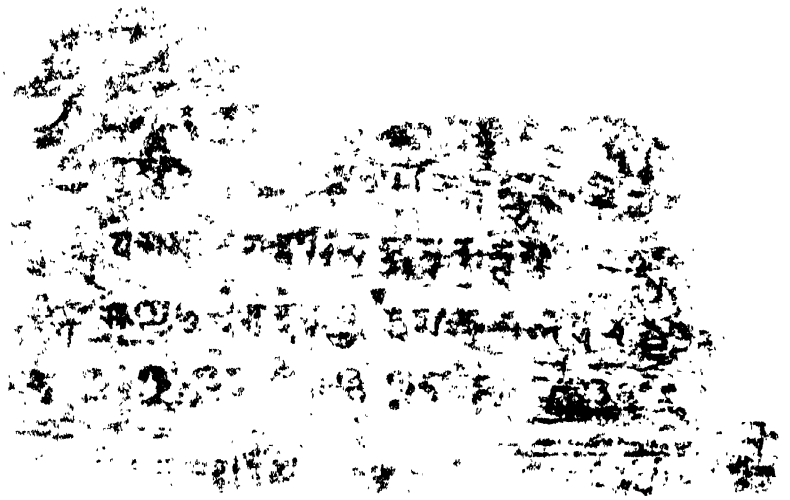

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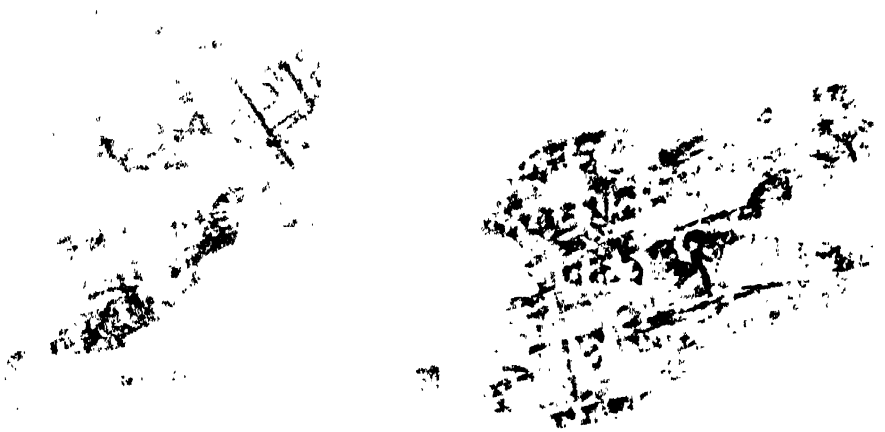
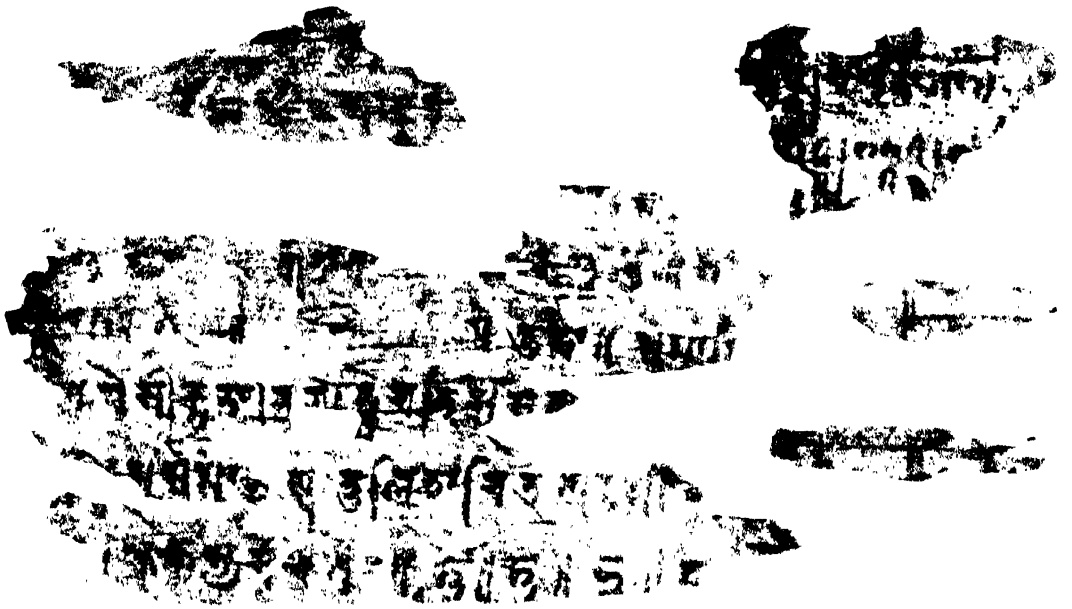


68 RECTO

Plate XLVI

69 RECTO

69 VERSO



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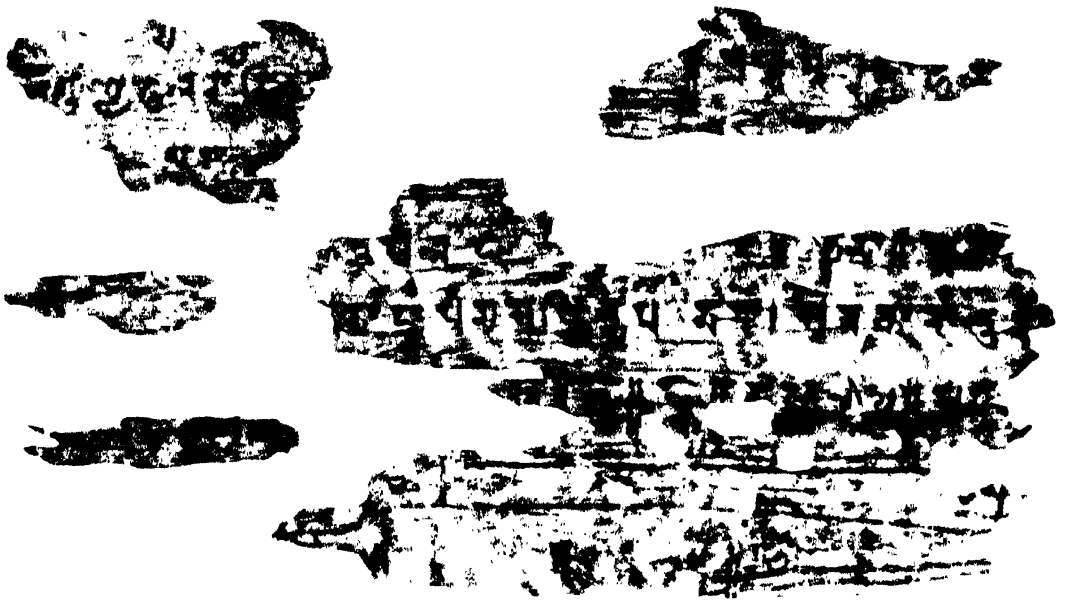
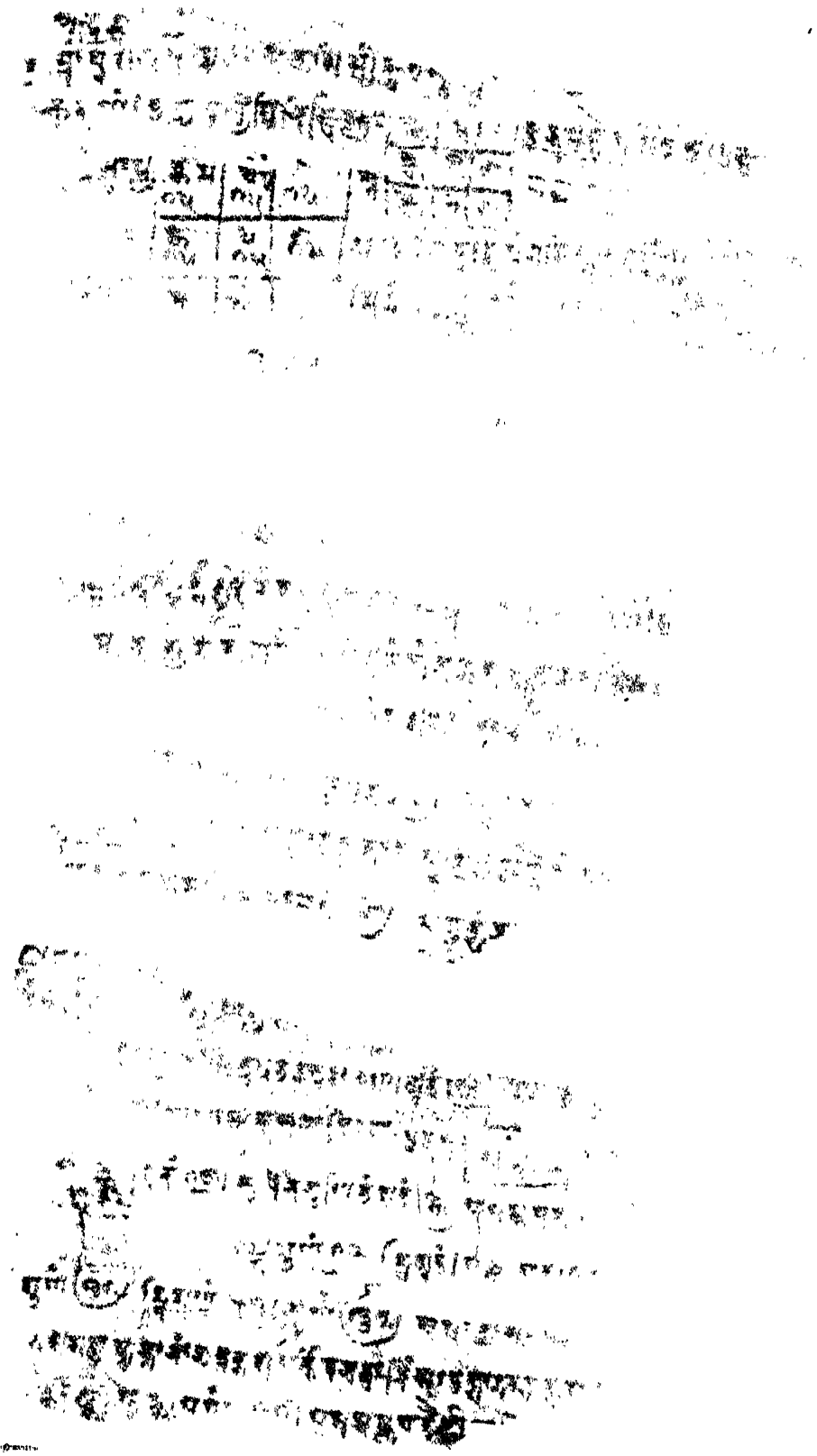


70 VERSO

Plat



RE ARRANGED FRAGMENTS

THE BAKHSHĀLĪ MANUSCRIPT AS PRESERVED IN THE BODLEIAN LIBRAR